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Application of magnetic anomaly data for depicting hydrogeological model of fractured aquifer: A case study from Kutasari, Purbalingga, Indonesia

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Abstract: Groundwater exploration generally uses the geoelectric method which faces difficulties in injecting electric current in hard rock environments, such as volcanic formations. Therefore, this study aimed to use magnetic anomaly data to obtain hydrogeological cross-section model of fractured aquifer in a volcanic rock complex in Kutasari District, Purbalingga Regency, Indonesia. Magnetic data were acquired at 768 points across the study area. The data processing followed the standard stages up to when local magnetic anomaly data were obtained and reduced to the poles. The results showed that First Horizontal Derivative (FHD) analysis indicated a maximum horizontal gradient pattern oriented from north to south. This gradient suggests a potential groundwater flow path through fractures and cracks in the volcanic rock complex. Fractures and cracks were interpreted to form an interconnected fractured aquifer system. Furthermore, forward modeling along AB and CD trajectories on the local anomaly contour map produced two hydrogeological cross-section models. Lithology interpretation showed that fractured aquifer was in the first and second layers. The first layer of vesicular andesite lava rocks was arranged with magnetic susceptibility values of 0.001134 and 0.001451 in cgs units. These rocks possess many cavities and have experienced extensive weathering. The second layer was composed of andesite lava rocks which have many cracks and fractures, with magnetic susceptibility values of 0.034200 and 0.048230 in cgs units. The third layer (not an aquifer) was made of dense and massive andesite lava with magnetic susceptibility values of 0.059500 and 0.059320 in cgs units. In conclusion, the results demonstrate the potential of fractured aquifer in Kutasari District area to support groundwater-based irrigation programs.

Keywords: Magnetic anomaly, hydrogeological modeling, fractured aquifer, hydrogeology, Kutasari District, Indonesia

INTRODUCTION

Groundwater refers to freshwater stored below the Earth surface, occupying the area between soil particles and fractures in rock formations. It plays a significant role in the water cycle and is an essential source of fresh water for irrigation, daily living, and industry. Groundwater is stored in aquifer composed of permeable rock, such as sand, that can hold and transmit water (Darwis, 2018). Amongst several geophysical methods for groundwater exploration, the geoelectric method is most widely used (Lubis, 2017) due to its ability to distinguish the aquifer from surrounding lithologies based on resistivity contrasts (Wahab *et al*., 2021). In volcanic rocks, data acquisition in geoelectric surveys often faces limitations due to poor electric current penetration into hard rocks. So, a magnetic survey is an alternative method for groundwater exploration in these areas (Oni *et al*., 2020). The magnetic method is particularly effective for detecting and mapping various volcanic rocks due to their mineral content with high magnetic susceptibility (Li & Fu, 2019). Magnetic susceptibility plays a crucial role in modeling and identifying subsurface rock formations or anomaly objects (Sehah *et al*., 2020). This parameter measures a material magnetism response level to an external magnetic field. In addition, magnetic field anomaly data used has high spatial resolution (Ritis & Chiappini, 2023). Filtering magnetic anomaly data helps to identify and locate fractures and cracks in volcanic rock formations (Terrone *et al*., 2021). In general, the structure resembles a boundary area between the volcanic rocks. Cracks and fractures are typically filled with rainwater to a certain depth, creating a fractured aquifer in the volcanic rock environment (Brito *et al*., 2020).

This study explored the Purwokerto-Purbalingga Groundwater Basin area, located along the southeastern slope of Slamet Volcano in Kutasari, Purbalingga, Indonesia. The basin area covers Banyumas to Banjarnegara areas, which is approximately 1,318.2 km^2 with a potential of 502.60

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million m^3 shallow and 9.70 million m^3 deep groundwater (Ramadhan, 2020). The area is estimated to be a groundwater route from the upper slopes of Slamet Volcano to the lowlands in the residential areas of Purwokerto city and Purbalingga city, Indonesia. Geologically, it comprises lahar and andesite lava deposits. Fractures and cavities are also oftenly found in volcanic rock complexes to act as aquifer with good capacity (Terrone *et al*., 2021). Generally, groundwater basin occupies a depression zone in central Java, between North and South Serayu Mountains. The basin is filled with alluvial deposits that presumably originate from sedimentary material created by rock weathering and erosion due to water flows from the surrounding mountain slopes. The material was deposited in the basin over thousands to millions of years, creating thick alluvial deposits that later transformed into aquifer (Bammelen, 1949).

Magnetic methods have limitations in interpretation due to the nature of magnetic dipoles. This limitation may lead to multiple possible interpretations of magnetic anomaly maps, making it challenging to model anomaly data, particularly in low latitude areas e.g. Indonesia (Zuo *et al*., 2021). Therefore, magnetic anomaly data requires special processing to reduce the effect of the dipole, facilitating modeling and interpretation. One effective filter for this purpose is the reduction to the pole, a transformation of magnetic anomaly data directed towards magnetic north pole. The filtering generates magnetic anomaly data similar to the conditions at the north pole, where the inclination and declination are considered 90° and 0° (Ansari & Ghorbani, 2009). Reducing magnetic anomaly data to the poles typically results in anomaly that have a monopole nature and a symmetrical pattern. As per the gravity anomaly contour map, the anomalous object is estimated to be located directly below the anomaly closure. Reduction to the Poles (RTP) filter is generally applied in analyzing magnetic anomaly data to identify iron ore (Sehah *et al*., 2020), geothermal systems (Daruwati, 2014), faults (Koesuma *et al*., 2024), and other structures.

This study focuses on characterizing the subsurface anomalous source (fractured aquifer). Although RTP magnetic anomaly effectively locates the anomaly, identifying the lithological boundaries requires First Horizontal Derivative (FHD) analysis (Daud *et al*., 2019). The highest horizontal derivative of magnetic anomaly data that arises from subsurface objects is typically concentrated along the edges of the subsurface object (Ansari & Ghorbani, 2009). Therefore, this method is beneficial for detecting and mapping lithological boundaries in a lateral direction. RTP magnetic anomaly data can be filtered using FHD. The maximum FHD occurs at points where the magnetization direction changes abruptly in the vertical direction, signaling a shift in the rock lithology. In a two-dimensional interpretation, the maximum FHD value typically forms peak or ridge patterns above areas that experience sudden changes in magnetization. This peak shows a lithological contact boundary, indicating the presence of faults, fractures, and cracks in the volcanic rock complexes (Oni *et al*., 2020). Groundwater in fractured aquifer is stored in fractures, joints, cracks, and cavities in rock masses. The availability is influenced by the characteristics and connectivity of these fractures (Chandra *et al*., 2019).

Geological review

The study area is comprised of the southeastern Slamet Volcano slope area, serving as a groundwater flow path to Purwokerto-Purbalingga Groundwater Basin. Geologically, the rock formation of Slamet Volcano can be divided into three rock formations including undifferentiated volcanic rocks (Qvs) consisting of lava, tuff, and volcanic breccia; lava of Slamet Volcano (Qvls) consisting of andesite lava with many cavities, cracks, and fractures, specifically on the eastern and southeastern slopes; Slamet Volcano lahar deposits (Qls) consisting of lava with andesite-basaltic volcanic rock boulders (Djuri *et al*., 1996). The present study focuses on the boundary area between the lava rock formation and lahar deposits of Slamet Volcano (Figure 1).

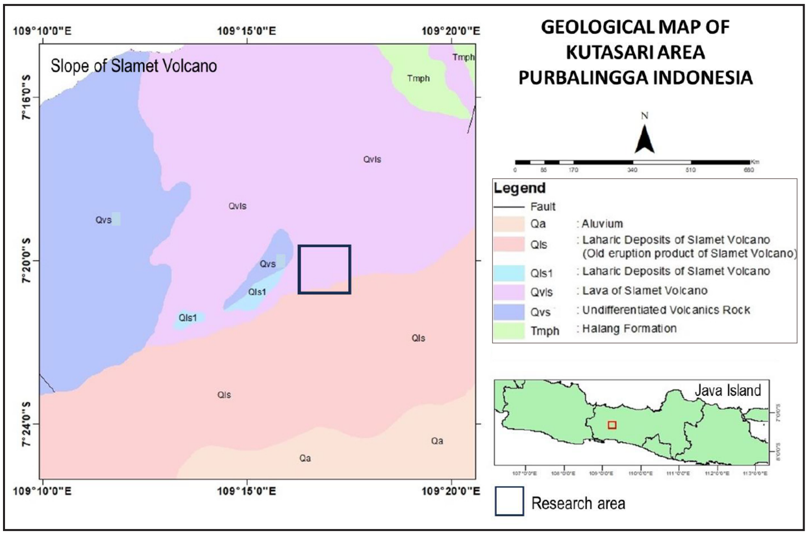


Figure 1: Geological map of the study area taken from the Purwokerto-Tegal Sheet (Djuri *et al*., 1996).

A groundwater discharge area is located to the southeast of Slamet Volcano, a significant part of Purwokerto-Purbalingga Groundwater Basin. The area occupies the Alluvium Formation (Qa), consisting of clay, silt, sand, gravel, and pebbles resulting from the deposition of the river and coastal materials (Djuri *et al*., 1996). This formation is composed of unconsolidated materials with spaces (pores) between the particles which allow water to infiltrate and be stored within the deposit.

Geomorphologically, the study area on the southeastern Slamet Volcano slope is part of a young volcano. These slopes are relatively gentle and not too steep compared to those at the west of Slamet Volcano, that is part of an old volcano (Prasetya & Gibran, 2024). In general, the slope influences the rise of several springs in this area. Groundwater flows through cracks and fractures in the rocks, moving from higher to lower elevations. Fractures and cracks are created in volcanic rocks by the eruption of Slamet Volcano or are formed in fault areas. Field observations show that groundwater originates from subsurface rock fractures on the lower slopes of Slamet Volcano, extending approximately 200 m and generally flowing in a north-to-south orientation (Iswahyudi *et al*., 2018). Several springs originate in fractured aquifers in the Situ Tirta Marta and Tuk Sirah lakes, located in the villages of Karangcegak and Candiwulan, respectively, in Kutasari District, Purbalingga Regency. These springs constitute an important, sustainable source of freshwater for humanity. They provide a naturally filtered and reliable water source, often providing communities and ecosystems with water for a long period of time.

Magnetic survey review

Magnetic survey is a geophysical method used for subsurface investigation. It is based on Earth magnetic field variations as observed at the surface (Telford *et al*., 1990). Magnetic field variations respond to the distribution of magnetic anomalous sources, including magnetized rocks, minerals, metals, and other subsurface geological structures (Maubana & Pedro, 2021). Magnetic field variations observed at the surface are in the background of Earth large magnetic field. In contrast, the observed magnetic anomaly is affected by variations in geological structures and rocks beneath the Earth surface (Alao *et al*., 2024). Some examples are volcanic rocks, iron ores, and sedimentary materials. In magnetic surveys, the focus is typically on magnetic anomaly which can be identified through spatial mapping of magnetic field values on Earth surface. It allows for the effective detection of subsurface anomaly sources (Butler *et al*., 2024). Magnetic anomaly are the fundamental variable in modeling subsurface geological structures. Therefore, related data can be effectively used to identify and understand the characteristics of geological objects, including detecting fractured aquifer in volcanic rock formations (Dumont *et al*., 2021).

Magnetic anomaly refers to a subsurface volume of rock that contains magnetic minerals. It originates from the geological structure or magnetized rocks and minerals distribution beneath Earth surface, resulting in a unique magnetic field. According to Telford *et al*. (1990), magnetic material volume can be considered a dipole (Figure 2). Magnetization magnitude of a material is influenced by Earth′s main magnetic field. Magnetic potential magnitude contained at a point in the rock volume can be expressed by Equation (1).

$$V(\vec{r}_0) = -C_m M\partial/\partial\alpha \int[dV/|\vec{r}_0 - \vec{r}|] \qquad (1)$$

Where M is magnetic dipole moment per unit volume, and C_m is a magnetic constant. Equation (2) defines magnitude of the material total magnetic induction (Telford *et al*., 1990).

$$\vec{B}(\vec{r}_0) = C_m \nabla\int_V \vec{M}(\vec{r}) \bullet\nabla\ [1/|\vec{r}_0 - \vec{r}|]dV \qquad (2)$$

Studies usually target magnetic anomaly ($\vec{B}(\vec{r}_0)$) which indicate variations in magnetization of subsurface minerals and rocks. The values can be used to model and interpret geological structures or subsurface formations. One application of magnetic surveying is to interpret fractured aquifer layers found in volcanic rock complexes (Fernandes *et al*., 2023).

METHODOLOGY

Area, time, and equipment

Data acquisition of magnetic field was conducted in the villages of Karangcegak, Sumingkir, and Candiwulan, located in Kutasari District of Purbalingga Regency, Indonesia. These villages lay along the groundwater pathway from the recharge to the discharge areas of Purwokerto-Purbalingga Basin. The width of the study area was 6.75 km^2, and the amount of magnetic data successfully

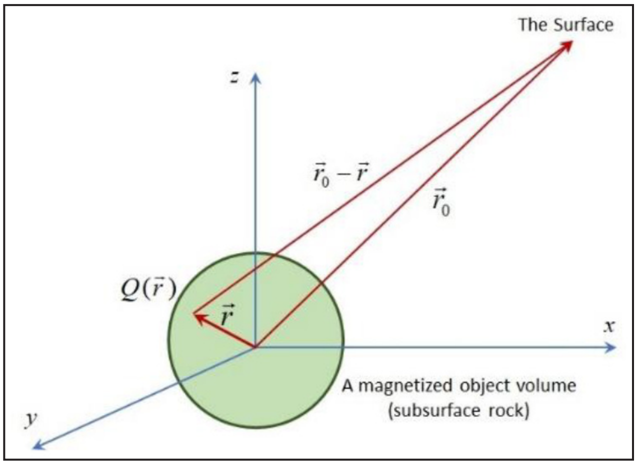


Figure 2: Illustration of magnetic anomaly from magnetized object volume beneath the Earth's surface (Telford *et al*., 1990).

acquired in the field was 768 points with a spatial resolution of 100 m. The study activities were conducted over 6 months, from June-November 2024. GSM-19T Proton Precession Magnetometer (accuracy of 0.05 nT) was used for data acquisition. Global Positioning System (GPS) was used to measure the geographic position of data points, and a navigation compass to orient magnetometer sensor towards North of Earth magnetic field. A Google Earth application was used for designing magnetic data acquisition in the field, while a personal computer was used for processing the data. Various other software also were used for processing and analyzing magnetic field anomaly data (Sehah *et al*., 2020).

Study procedure

The results obtained from the field work include total magnetic intensity data distributed on the topographical surface, according to the spread of location points in the survey design (Figure 3). Daily and IGRF corrections were applied to eliminate daily variations and the contribution of Earth main magnetic field to obtain total anomaly data. IGRF is the International Geomagnetic Reference Field, a standard mathematical description of the main magnetic field (Alken *et al*., 2021). Total magnetic anomaly data were generated after some corrections were made. In the next step, the total magnetic anomaly data were transformed from topographic to the horizontal surface (Blakely, 1995) and separated from the regional anomaly data (Telford *et al*., 1990 and Kebede *et al*., 2020). After processing and separation, local magnetic anomaly data were obtained (Sehah *et al*., 2020). These data are residual anomaly obtained after correcting Earth main and external magnetic field, as well as regional anomaly against the measured total magnetic field data (Berguig *et al*., 2016). Generally, local magnetic anomaly data shows shallow geologic structures, including subsurface rocks (Meyer, 2013).

To model and interpret complex local magnetic anomaly data due to the presence of different magnetic dipoles, RTP was used to simplify and increase the accuracy (Hayatudeen *et al*., 2021). RTP local magnetic anomaly data were analyzed using FHD method to obtain the maximum horizontal gradient patterns (Daud *et al*., 2019). These patterns indicate differences in magnetic susceptibility value between different subsurface anomalous objects, suggesting the presence of lithological boundaries (cracks, fractures, faults, and other structures) or edges (Rosid & Siregar, 2017). The presence of cracks and fractures filled with groundwater generated in volcanic rocks was determined using the maximum patterns of FHD contour. Furthermore, Equation (3) was applied to calculate or determine FHD value ($h(x,y)$) from magnetic anomaly data ($\Delta B(x,y)$), which was spatially dispersed (x,y) on the horizontal surface (Blakely, 1995).

$$h(x,y) = [(\partial(\Delta B(x,y))/\partial x)^2 + (\partial(\Delta B(x,y))/\partial y)^2\ ]^{(1/2)} \quad (3)$$

RESULTS AND DISCUSSION

Results of magnetic data processing

This section focuses solely on the data processing results based on the procedures outlined in the methods. Following the correction and reduction stages, magnetic data processing produced the total magnetic anomaly data. Considering that the target, namely fractured aquifer, originated from shallow subsurface anomaly sources (Alao *et al*., 2024), data from deep and wide sources must be separated (Ilapadila *et al*., 2019). Consequently, an upward continuation method to a height of 2000 m was used to separate magnetic anomaly data. The separation resulted in local magnetic anomaly data (Figure 4). The dominance of negative values in local magnetic anomaly indicates the presence of a fractured aquifer in volcanic rock complexes (Table 1). Generally, a fractured aquifer has good groundwater potential.

Based on geological information, lithology of the study area was dominated by andesite lava rock (Djuri *et al*.,

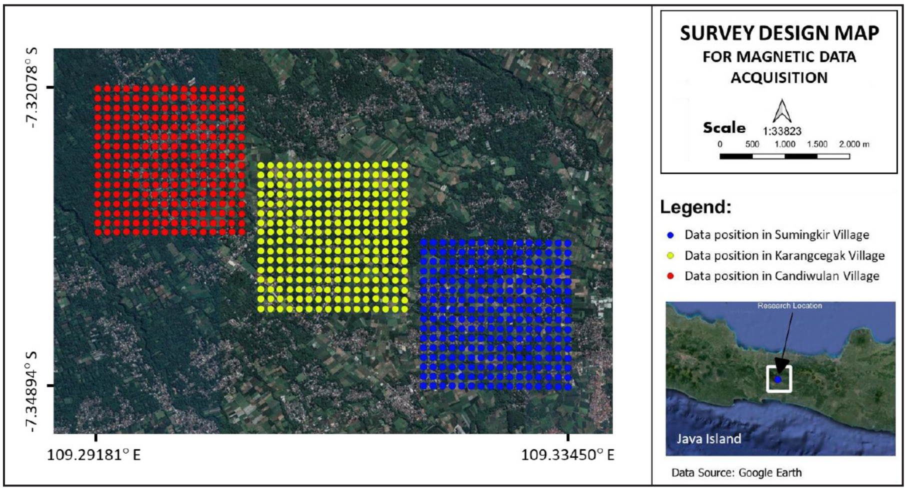
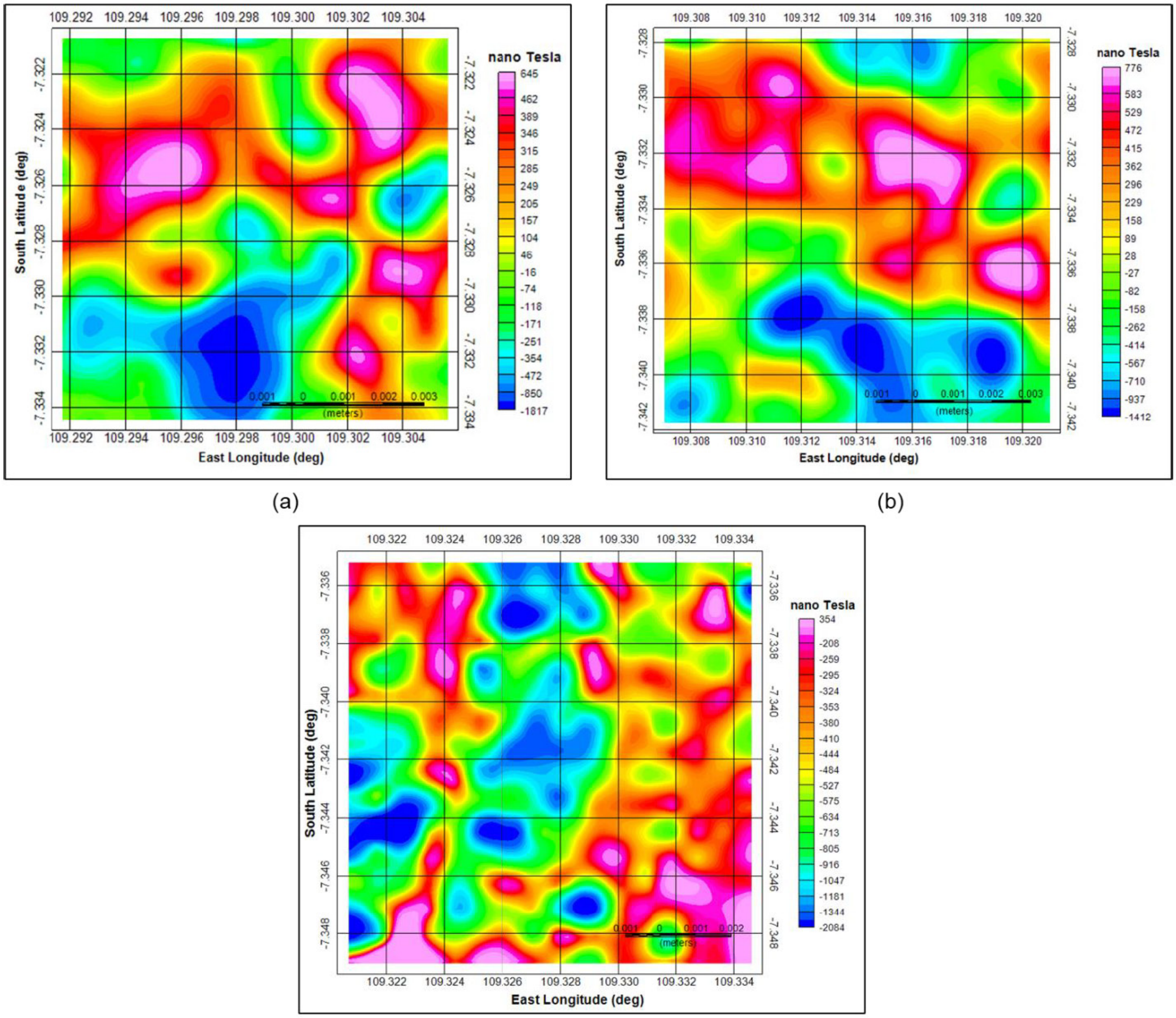


Figure 3: Study design map for acquisition of magnetic field data in the study area.



(a)

(b)

(c)

Figure 4: Local magnetic anomaly contour maps for several areas, including (a) Candiwulan Village, (b) Karangcegak Village, and (c) Sumingkir Village, Kutasari District, Purbalingga Regency.

Table 1: Results of magnetic field data processing in the study area to obtain local magnetic anomaly data.

No.	Study Area	Magnetic Anomaly Interval (nT)		
		Total	Regional	Local
1	Candiwulan Village Area	-1561.12 – 899.47	257.08 – 257.63	-1818.21 – 642.01
2	Karangcegak Village Area	-1214.20 – 970.25	193.81 – 194.44	-1408.16 – 775.95
3	Sumingkir Village Area	-655.15 – 1766.51	301.13 – 301.32	-2067.76 – 353.96

1996), which typically has a high magnetic susceptibility value. The large local magnetic anomaly interval (Table 1), indicates a high magnetic susceptibility (Sehah *et al*., 2020). Apart from andesite rocks, several others in the study area are believed to have high magnetic susceptibility values, such as andesite tuff, basaltic, and volcanic breccia. The range of magnetic susceptibility values for volcanic breccia is classified as medium to high (Li & Fu, 2019). The andesite lava that dominates the study area has several cavities, fractures, and cracks (Iswahyudi *et al*., 2018). Groundwater originating from the upper slope can easily fill the spaces in the rock, thereby contributing to a reduction in total magnetic susceptibility (Oni *et al*., 2020). Fractures and cracks in volcanic rocks often occur due to shrinkage of the body in cold conditions and expansion in hot conditions (Heinze, 2024). Meanwhile, cavities in volcanic rocks are formed during the solidification of lava on the Earth's surface. They help geologists to understand the cooling history of extrusive volcanic rocks because lava contains large amounts of dissolved gases that are released as lava hardens.

Results of magnetic data filtering

Reduction to the poles in the context of magnetic anomaly refers to the mathematical process applied to filter data from the effect of the inclination and declination of Earth′s magnetic field. Different inclinations and declinations influenced magnetic anomaly values were recorded when magnetic data were acquired at a specific area. Magnetic field was non-uniform and tilted due to varying inclination and declination, resulting in magnetic anomaly in low-latitude areas that often appeared as asymmetrical patterns. The differences distort the observed anomaly, mainly when the source is not precisely at the area. The local anomaly demonstrates a significant high and low pattern with a less symmetrical shape (Figure 4). Anomalous contour patterns make it difficult to identify sources of anomaly beneath the Earth surface (Yolanda *et al*., 2018). Therefore, reducing magnetic anomaly to the poles helps better visualize and understand the sources by compensating for the inclination and declination at the observation point (Rajagopalan, 2003).

Applying RTP method requires adjusting the orientation of Earth magnetic field with fixed inclination and declination angles towards the poles, which are 90° and 0°, respectively (Blakely, 1995). Changes in these parameters will cause the asymmetrical pattern on the contour map to become symmetric. An asymmetrical anomaly profile indicates the area of the source below the peak of the curve or in the middle of the closure. Therefore, RTP method has the potential to show the correlation between local magnetic anomaly data and local geological information through mathematical modeling (Table 2) (Rusman *et al*., 2023). Meanwhile, RTP magnetic anomaly contour maps for the entire study area (Figure 5). Meanwhile, the RTP magnetic anomaly contour maps for the entire study area show that the green-to-blue contours correspond to regions with low RTP values. Below such areas, fractured aquifers are typically found.

FHD of magnetic field anomaly is a geophysical method commonly used to improve the interpretation of sources. It shows lateral changes in magnetic field, facilitating the identification of the edges of anomaly sources such as faults, fractures, dikes, or geological boundaries (Banji *et al*., 2019). The obtained values from FHD contour map reflect variations in the values between points, with clear peaks visible at the boundaries or edges of the anomaly objects (Ekwok *et al*., 2022). However, when FHD filter was applied for two-dimensional modeling, the maximum values of the horizontal derivative were likely found along narrow ridges or peaks corresponding to sudden magnetization changes at the edge of the anomaly object (Blakely, 1995). Therefore, FHD filter application produced information on the lithological boundaries of subsurface rocks based on variations in magnetization along lateral direction. FHD is often shown as a map, where high gradient zones along narrow ridges correspond to the boundaries of magnetic anomaly source, aiding quantitative interpretation. The complete calculation results of FHD on RTP local magnetic anomaly data (Table 3) show variations in FHD values. These variations are also evident on the FHD contour maps for the entire study area (Figure 6). The pink-to-red contour pattern (indicating high FHD values) suggests the presence of fractured aquifer that have limited water storage and flow capacity but can be significant when large enough and interconnected. The distribution of groundwater in the rock is uneven, generally adjusting the number and size of fractures.

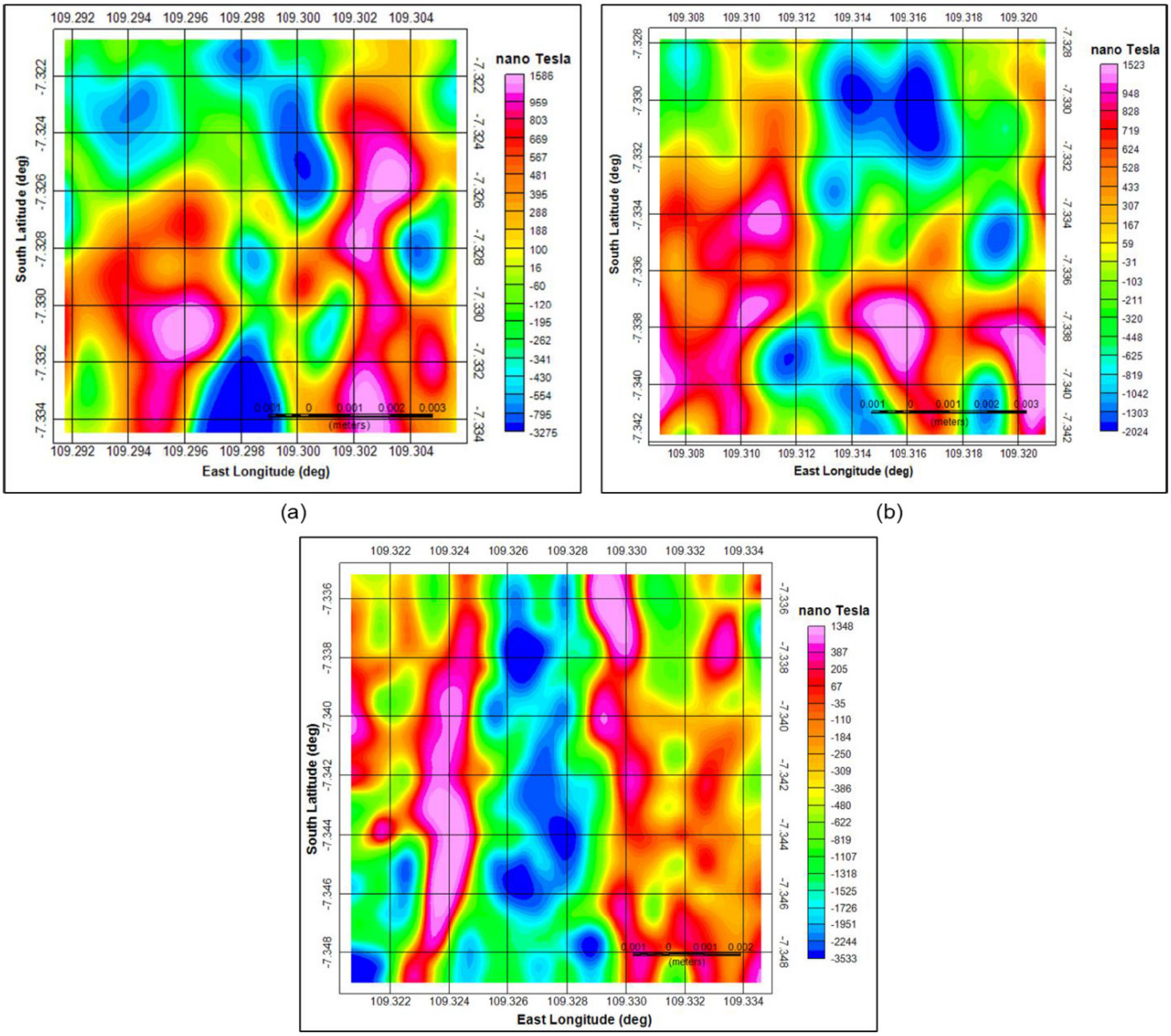
FHD contour pattern identified in the study area shows a pattern of north-south continuity or straightness. This pattern is in accordance with the results of field observations, showing rock fractures on the lower slopes

Table 2: Calculation results of RTP local magnetic anomaly in the study area.

No.	Study Area	Angle (degree)		Local Magnetic Anomaly (nT)	
		Inclination	Declination	Before RTP	After RTP
1	Candiwulan Village Area	-31.1560	0.5798	-1818.21 – 642.01	-3307.42 – 1591.85
2	Karangcegak Village Area	-31.1805	0.5839	-1408.16 – 775.95	-2022.49 – 1522.89
3	Sumingkir Village Area	-31.1950	0.5821	-2067.76 – 353.96	-3553.65 – 1323.60

Table 3: Calculation results of FHD on RTP local magnetic anomaly data.

No.	Study Area	RTP Local Magnetic Anomaly (nT)	FHD (nT/m)
1	Candiwulan Village Area	-3307.42 – 1591.85	0.161 – 21.930
2	Karangcegak Village Area	-2022.49 – 1522.89	0.076 – 21.441
3	Sumingkir Village Area	-3553.65 – 1323.60	0.088 – 32.834



(a) (b)

(c)

Figure 5: Local magnetic anomaly contour maps (reduced to the poles) for several areas, including (a) Candiwulan Village, (b) Karangcegak Village, and (c) Sumingkir Village, Kutasari District, Purbalingga Regency.

of Slamet Volcano, which extend from North to south (Iswahyudi *et al*., 2018). In FHD map (Figure 6), the maximum horizontal gradient pattern indicates the area of sharp changes in magnetic field, which are interpreted as fractures or cracks in volcanic rocks. In Candiwulan and Karangcegak village areas, these fractures or cracks are cut by topography, allowing the rise of groundwater flows up to the surface as springs. There are several springs fill Tuk Sirah and Situ Tirta Marta Lakes in these villages. Another possibility is that the geological structure is a weak zone that allows groundwater flow from the subsurface to penetrate the surface. The maximum horizontal gradient pattern is shown in red to pink contour colors, which serves as a qualitative method to illustrate potential fracture and crack pathways in volcanic rocks. Typically, fractures and cracks occur when lava cools and hardens or due to tectonic activity and other external forces (Liu *et al*., 2022). In several cases, fractures or cracks can develop into wide, deep structures in volcanic rock environments. The formation of fractures and cracks in volcanic rocks originates from various geological processes related to stress in Earth crust, but this is generally triggered by the rock inability to withstand tremendous pressure (Liu *et al*., 2022). This occurs when the pressure the rock receives exceeds the internal strength, causing cracks, collapse, or shifting.

Results of magnetic data modeling

The modeling target was the cross-section of fractured aquifer in a volcanic rock complex, hence, the two-dimensional forward modeling method was used (Kebede *et al*., 2021). This was carried out along Trajectories AB and CD on RTP local magnetic anomaly contour maps, with

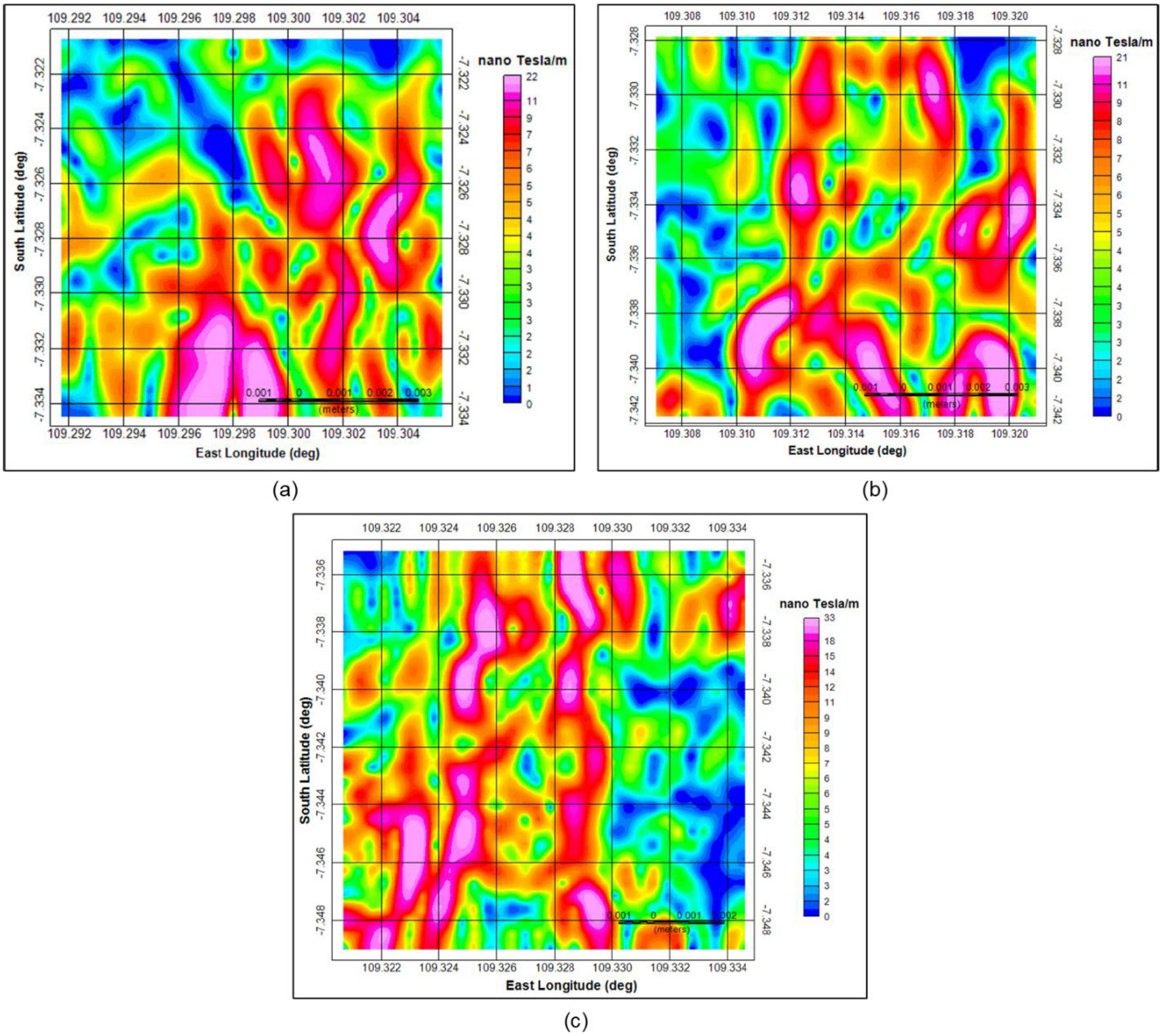


Figure 6: First horizontal derivative contour maps of RTP local magnetic anomaly for several areas, including (a) Candiwulan Village, (b) Karangcegak Village, and (c) Sumingkir Village, Kutasari District, Purbalingga Regency. A pink-to-red contour pattern (high FHD value) indicates fractured aquifer's presence.

Karangcegak village area as a case study. The evaluation of the subsurface geological structure is intriguing after Karangcegak village was selected for its widespread fractured groundwater manifestations. Trajectories of AB and CD can be placed on the contour map, which serves as the target around Tirta Marta Lake (Figure 7). RTP local magnetic anomaly data were taken along these trajectories at evenly spaced intervals. These observed magnetic anomaly data served as a comparative reference against the calculated anomaly data in forward modeling (Essa *et al*., 2023). Forward modeling generally can be achieved by calculating magnetic response based on the initial model and comparing with observed data. When a strong concordance is observed between the observed and calculated curves, the resulting anomaly model can be considered a representation of the actual subsurface structure (Sehah *et al*., 2020). A match between observed and calculated anomaly data is achieved by making iterative adjustments to the model parameters.

The anomalous sources modeling results showed three subsurface rock layers (Table 4, 5) with magnetic susceptibility values along trajectory AB ranging from 0.001134-0.05950 in cgs unit (Figure 8). Meanwhile, rock magnetic susceptibility values along trajectory CD ranged from 0.001451-0.059320 in cgs (Figure 9). The first rock layer consisted of vesicular andesite lava rock, characterized by numerous cavities (Iswahyudi *et al*., 2018), most of which have experienced weathering (Sehah *et al*., 2021), allowing groundwater to penetrate easily. Andesite lava deposits

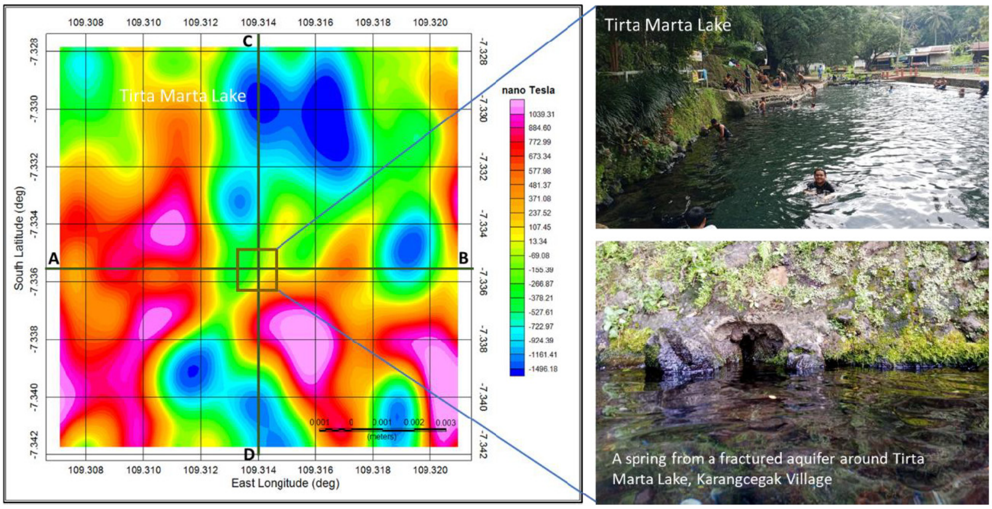


Figure 7: Trajectories AB and CD for 2D-modeling of RTP local magnetic anomaly data; one is through the Situ Tirta Marta Lake area.

Table 4: Interpretation of lithology and hydrogeology of subsurface rocks for trajectory AB.

No.	Depth of Model (m)	Magnetic Susceptibility Values (cgs unit)	Interpretation	
			Lithology	Hydrogeology
1	0 – 152	0.001134	Vesicular andesite lava rocks with many cavities, most of which have weathered	Shallow fractured aquifer
2	152 – 304	0.034200	Andesite lava rocks with many cracks and fractures	Deep fractured aquifer
3	304 – 500	0.059500	Massive andesite lava rocks (relatively more denser)	Nonaquifer

Table 5: Interpretation of lithology and hydrogeology of subsurface rocks for trajectory CD.

No.	Depth of Model (m)	Magnetic Susceptibility Values (cgs unit)	Interpretation	
			Lithology	Hydrogeology
1	0 – 170	0.001451	Vesicular andesite lava rocks with many cavities, most of which have weathered	Shallow fractured aquifer
2	171 – 300	0.048230	Andesite lava rocks with many cracks and fractures	Deep fractured aquifer
3	301 – 500	0.059320	Massive andesite lava rocks (relatively more denser)	Nonaquifer

containing many cracks and fractures are estimated to be in the second rock layer (Iswahyudi *et al*., 2018). Meanwhile, the third layer was estimated to be massive and relatively denser andesite lava rock of Slamet Volcano. When the volcanic rocks are filled with groundwater up to a water-saturated condition, magnetic susceptibility decreases, forming a low magnetic anomaly (Aïfa *et al*., 2014). Based on the interpretation results, fractured aquifer was estimated to be in the first and second rock layers. The first layer functions as a shallow aquifer, while the second layer possibly is a deep aquifer. Tirta Marta Lake, located in the middle part of the study area, was estimated to receive groundwater from the first rock layer based on modeling results on AB and CD trajectories. Fractured groundwater flows from the upper slopes of Slamet Volcano through cracks or fractures in rocks below Earth's surface. These fractures enable the movement of water through otherwise impermeable rock formations, forming what are known as fractured aquifers.

Analysis and discussion

The study area, the northern part of the Purwokerto-Purbalingga Groundwater Basin is a connecting zone

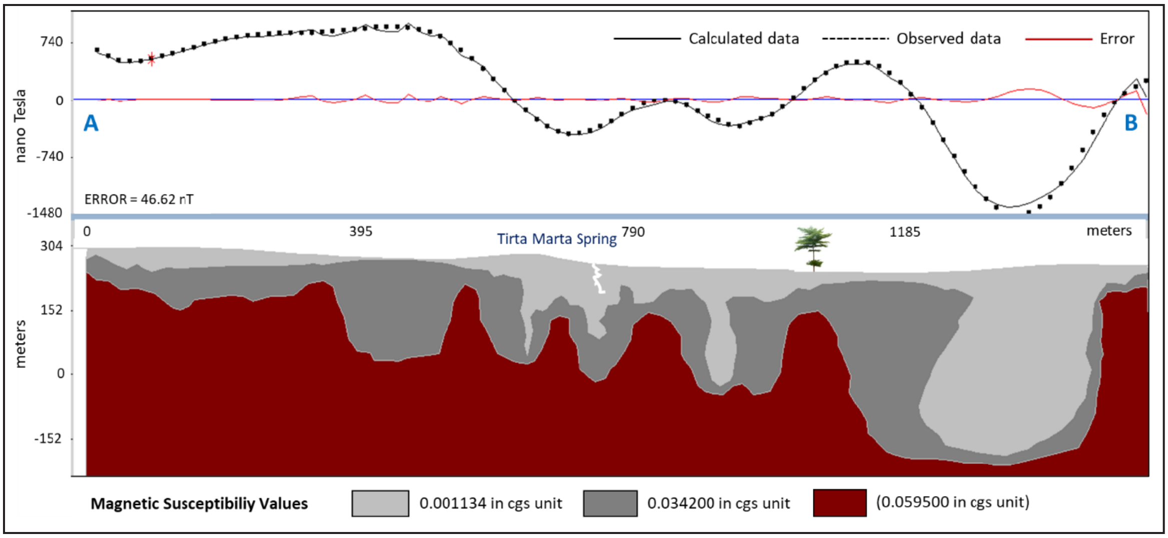


Figure 8: Result of 2D hydrogeological modeling on RTP local magnetic anomaly data along trajectory AB.

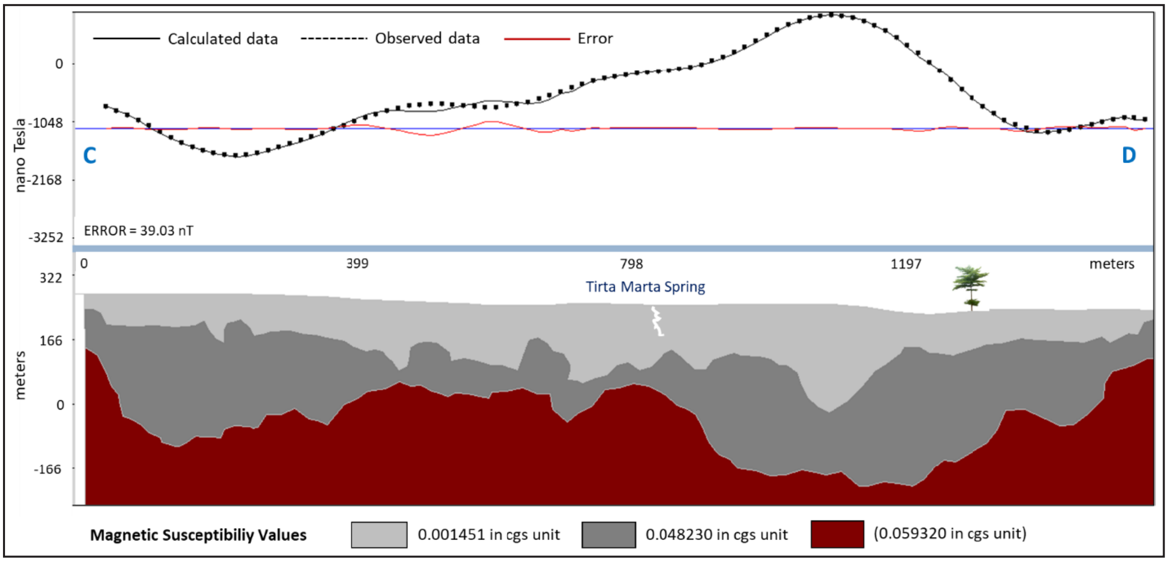


Figure 9: Result of 2D hydrogeological modeling on RTP local magnetic anomaly data along trajectory CD.

between the groundwater recharge area on the upper slopes of Slamet Volcano and the discharge area in the lowlands. This lowland is located above a geological basin containing alluvial deposits and stretches from Purwokerto City to Purbalingga Regency areas, surrounded by highlands and mountains (Ramadhan, 2020). Referring to the topographical conditions of the study area, groundwater is estimated to flow from the higher northwest and northeast to the lower southeast and southwest areas. Groundwater originating from the recharge area moves through fractured aquifer in the volcanic rock complexes along the slopes of Slamet Volcano, and fills the aquifer layer in the basin. The slopes have unique characteristics that affect hydrological system, such as the appearance of volcanic rocks with varying porosity and permeability. Solidified lava often leads to cavities, fractures, and cracks, increasing the volcanic rock porosity and permeability resulting in aquifer (Sutawidjaja & Sukhyar, 2009).

Shallow aquifer are estimated to develop in rock layer close to the surface of the Slamet Volcano slope, where groundwater is stored in the spaces or cracks under unconfined conditions. The modeling results (Figures 8, 9) show that this rock layer consists of vesicular andesite lava rocks, containing numerous cavities, and the majority have experienced weathering. Deep aquifer are estimated to be in the next layer, typically at greater depths. Modeling results estimated that the area of the deep aquifer was in an andesite lava formation characterized by numerous cracks and fractures. Meanwhile, the lowest rock layer was estimated to consist of massive andesite lava with a

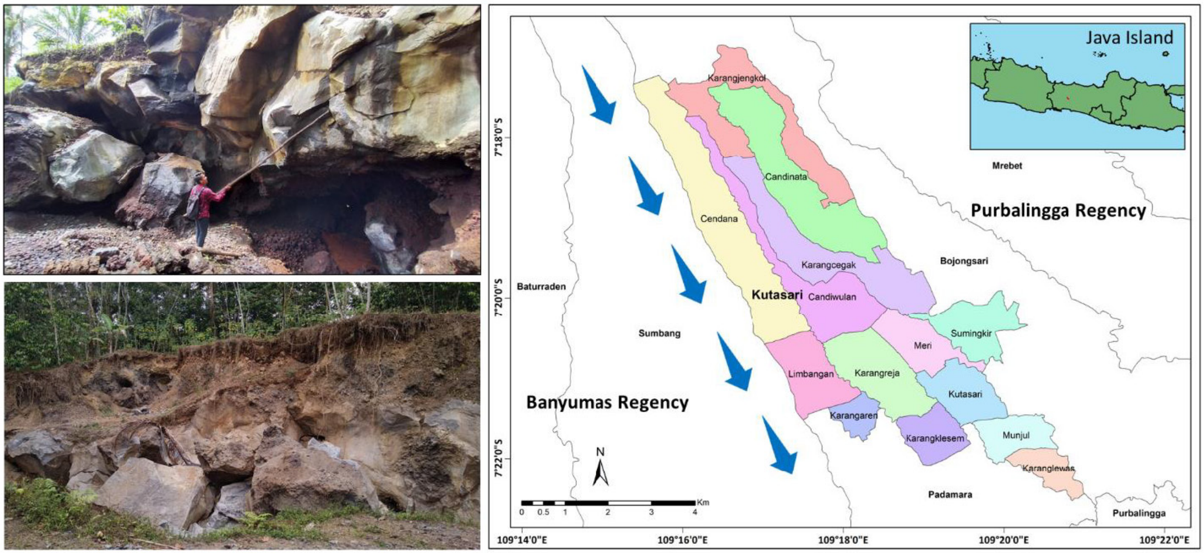


Figure 10: The appearance of andesite lava rock outcrops with large cracks and fractures and the estimated direction of groundwater flow in the study area (marked with blue arrows).

relatively higher density. It was dark grey and aphanitic, with a layered structure, and was estimated to have many fractures (Iswahyudi *et al*., 2018). The volcanic rock environment typically consists of three layers, namely weathering at the top, fracture in the middle, and massive at the bottom (Cabria, 2015).

Water sources on volcanic slopes generally originate from rainfall infiltration, considering that rainfall on the upper slopes of Slamet Volcano is relatively very high, reaching 8,134 mm per year (Purwanto, 2023). Aside from rainfall, the content of groundwater in this aquifer was also affected by the connectivity and characteristics of fractures and cracks (Ringel *et al*., 2024). These supporting data, in accordance with the maximum FHD pattern (Figure 6) indicates the presence of interconnected fractures and cracks in the villages. A great pattern may indicate wide fractures or cracks and vice versa. The study area, characterized by numerous cavities, cracks, and fractures, allows groundwater to fill and flow through layers of volcanic rock, creating productive fractured aquifer (Ramadhan, 2020). This type of fractured rock is usually called a joint which refers generally to cracks and fractures in volcanic rock complexes (typically open) (Razali & Roslan, 2019). The opening structures allow groundwater to fill and flow down to a lower area, namely the Purwokerto-Purbalingga Groundwater Basin discharge area. The appearance of andesite lava rock outcrops in all villages with large cracks and estimated groundwater flow in the study area are shown in Figure 10.

The flow of groundwater in fractured rock can be affected by several factors, including the size and depth of the opening, the length, the relationship between fractures and cracks, as well as the amount of rainwater infiltration into the rock and soil in the recharge area (Tran & Matthai, 2021). In the recharge areas, water seeps into the ground through spaces between grains, cracks, and fractures in the volcanic rock. Through interconnected fractures and cracks, groundwater contained in the rock moves and flows along the area slopes. This process continues under the influence of a gravity field to a certain depth where the groundwater meets the impermeable rock layer, which is massive andesite lava rocks. Consequently, groundwater flow tends to be relatively horizontal up to the discharge area (Brito *et al*., 2020). Fractured aquifer have a significant role in hydrogeological system due to the function as groundwater primary sources in areas where more porous sedimentary aquifer are unavailable. Therefore, studying fractured aquifer is essential, specifically in understanding the movement pattern and storage of groundwater in areas dominated by hard rocks such as andesite.

Studies related to hydrogeological model of fractured aquifer in volcanic rock complexes are very significant in the planning and managing of groundwater resources to maintain the balance of the ecosystem on the slopes of Slamet Volcano (Suwarno *et al*., 2022). It is the primary source of groundwater and surface water for the community living around the foot of Slamet Volcano. Conservation efforts on the eastern and southern slope regions are significant in maintaining the sustainability of groundwater sources. The forest in this area also functions as a good rainwater catchment zone and maintains stable groundwater availability (Glanville *et al*., 2023). Although groundwater availability is relatively abundant, challenges such as land use changes, deforestation, and volcanic activity can affect both its quality and quantity in the study area. To maintain optimal conditions, integrated and sustainable aquifer management, as well as forest protection, are critical for monitoring groundwater catchments. Effective management of fractured aquifers is essential for ensuring a sustainable fresh water supply (Meles *et al*., 2024), particularly for groundwater availability in Purwokerto-Purbalingga Groundwater Basin and its surrounding areas. Fractured groundwater plays an important role, specifically in areas with limited conventional aquifer, such as Kutasari District.

Geophysical interpretation is crucial in natural resource exploration, disaster mitigation, and geological studies. However, it is essential to recognize and address the limitations by combining with several other methods, using supporting data, and conducting field validation. The complex geological structure in the study area can distort data, complicating interpretation. Therefore, the results obtained from magnetic methods must be validated with geological or drilling information to improve accuracy. Comparing the results with other geophysical methods is very important to enhance the validity. Future studies should use seismic reflection methods for fractured aquifer to provide high resolution in shallow areas. Unlike geoelectric, the seismic reflection method is not constrained by the difficulty of injecting current into hard rocks. This study is particularly valuable for groundwater exploration in volcanic regions, where hard rocks dominate and groundwater is primarily found within fractures. Rock fractures serve as aquifers that can be used for domestic use, irrigation, and local industrial water supply.

CONCLUSION

This study proposed to develop hydrogeological model of fractured aquifer in volcanic rocks located in the Kutasari District area of Purbalingga Regency, Indonesia, using magnetic anomaly data. These data were collected in the field using the Proton Precession Magnetometer (PPM) at 768 locations spread in the study area and arranged regularly with a spatial resolution of 100 m. Magnetic data processing comprises several steps including correction, reduction, filtering, and separation. These processes resulted in the local magnetic anomaly data, representing local and near-surface geological structures. Local magnetic anomaly data obtained for Candiwulan, Karangcegak, and Sumingkir villages were -1818.208 – 642.012 nT, -408.160 – 775.946 nT, and -2067.760 – 353.964 nT, respectively. Furthermore, the local magnetic anomaly data were reduced to the poles to minimize magnetic dipole in data interpretation and improve the clarity of the subsurface anomaly source.

FHD analysis results of RTP local magnetic anomaly data showed maximum patterns in a relatively north-south direction. FHD patterns indicate crack and fracture pathways in volcanic rock complexes. The modeling results along AB and CD Trajectories produced hydrogeological model consisting of three subsurface rock layers with varying magnetic susceptibility values. The first layer showed vesicular andesite lava deposits characterized by many cavities that have experienced weathering with magnetic susceptibility values of 0.001134 and 0.001451 in cgs units. The second layer consisted of andesite lava characterized by many cracks and fractures with magnetic susceptibility values of 0.034200 and 0.048230 in cgs units. The third layer was interpreted as dense and massive andesite lava with magnetic susceptibility contrasts of 0.059500 and 0.059320 in cgs units. Based on the results, fractured aquifer were in the first and second layers. This study on fractured groundwater exploration is quite relevant and significant, specifically in volcanic areas with hard rocks. Fractured groundwater is the only accessible water source that can be used for irrigation, local industry, and other living needs.

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AUTHORS CONTRIBUTION

S was the first author who performed data acquisition, modeling, data interpretation, and prepared the manuscript. ANA is the co-author who assisted with data acquisition and wrote the manuscript. H is the co-author who assisted with data acquisition and data processing. FRS and MR are co-authors (students) who assisted with data acquisition and data processing. RFA and LSPB are co-authors who assisted wrote the manuscript.

CONFLICT OF INTEREST STATEMENT

The authors declare no affiliation in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this manuscript.

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