Numerical solution of Fuzzy Differential Equation Using Improved Runge-Kutta Method

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***Abstract*— In this paper a fuzzy Improved Runge-Kutta method for solving first-order fuzzy differential equations is proposed. The scheme is two step in nature and is based on the Improved Runge-kutta method for solving ordinary differential equations. Here, the fourth order method with three stages is explained. In this method some new parameter be exploited to increase the accuracy in comparison with other same stage existing methods. The convergence of the method is proven, and several numerical examples are experienced to illustrate the effectiveness of the method.**

***Keywords- Fuzzy Improved Runge-Kutta method, Fuzzy differential equations, Two step method, Improved Runge-Kutta method,* Seikkala derivative.**

Introduction

Fuzzy Differential Equations (FDEs) are used for modelling the problems in science and engineering. Most of the problems require the solution of FDEs which satisfied fuzzy initial conditions. The concept of fuzzy derivative was first introduced by Chang and Zadeh [1], later Dubois and Prade [2] proposed the extension principle for solving FDEs. It is difficult to find the exact solution of FDEs therefore several numerical methods were developed to address this problem. Abbasbandy and Allahviranloo [3] developed numerical algorithm for solving fuzzy differential equations based on Seikkala's work [4]. Ahmad and Hasan [5] presented a new fuzzy version of Euler's method for solving FDEs with fuzzy initial values. In this paper the Improved Runge-kutta method of order four with 3-stages given by Rabiei et al in [6] is developed for solving first order fuzzy initial value problems.

In sections 1 and 2, some basic definitions and theorem on FDEs are given. In section 3, Fuzzy Improved Runge-Kutta method of order four with three stages (FIRK4) is proposed and numerical examples to illustrate the efficiency of new method are given in section 4.

I. PRELIMINARIES

Fuzzy set is a generalization of a classical set that allows membership function to take any value in the unit interval [0, 1]. The formal definition of a fuzzy set is as follows:

Definition 1: (see [1]) Let Ω be a universal set. A fuzzy set A in Ω is defined by a membership function $A(t)$ that maps every element in Ω to the unit interval [0, 1]. A fuzzy set A in Ω may also be presented as a set of ordered pairs of a generic element t and its membership value, as shown in the following equation:

$$A = \{(t, A(t)) | t \in \Omega\}$$

Definition 2: (see [1]) Let A be a fuzzy set defined in Ω. The support of A is the crisp set of all elements in Ω such that the membership function of A is non-zero, that is,

$$\sup p(A) = \{t \in \Omega \mid A(t) > 0\}.$$

Definition 3: (see [7]) Let A be a fuzzy set defined in Ω by membership function $A(t): \Omega \to [0,1]$. Let us denote by $\mathbb{R}_{\mathbb{F}}$ the class of fuzzy subsets of the real axes (i.e. $A : \mathbb{R} \to [0,1]$) Satisfying the following properties:

1)) $\forall A \in \mathbb{R}_{\mathbb{F}}$, A is normal, that is there exists $t_0 \in \mathbb{R}$ such that $A(t_0) = 1$;
2)) $\forall A \in \mathbb{R}_{\mathbb{F}}$, A is convex, that is for all $t, y \in \mathbb{R}$ and $0 \le \lambda \le 1$, it holds that
$$A(\lambda t + (1-\lambda)y) \ge \min(A(t), A(y));$$
3)) $\forall A \in \mathbb{R}_{\mathbb{F}}$, A is upper semi-continuous on $\mathbb{R}$, that is for any $t_0 \in \mathbb{R}$, it holds that $A(t_0) \ge \lim_{t \to t_0^{\pm}} A(t)$
4)) $[A]^0 = cl\{t \in \mathbb{R} \mid A(t) \ge 0\}$ is a compact, where $cl(U)$ denotes the closure of subset *U*.

Then $\mathbb{R}_{\mathbb{F}}$ is called the space of fuzzy members. Obviously $\mathbb{R} \subset \mathbb{R}_{\mathbb{F}}$.

Definition 4: (see [1]) Let A be a fuzzy set defined in $\mathbb{R}_{\mathbb{F}}$. The r cut of A is the crisp set $[A]^r$ that contains all elements in $\mathbb{R}$ such that the membership values of A is greater than or equal to r , that is

$$[A]^r = \{t \in \mathbb{R} \mid A(t) \ge r\}, \quad r \in (0,1],$$
$$[A]^0 = cl\{t \in \mathbb{R} \mid A(t) > 0\} \ .$$

Definition 5: (see [7]) Let $D : \mathbb{R}_{\mathbb{F}} \times \mathbb{R}_{\mathbb{F}} \to \mathbb{R}_{+} \bigcup \{0\}$, $D(u,v) = Sup_{r\in[0\,1]} \max\{|u_1(r)-v_1(r)|, |u_2(r)-v_2(r)|\}$ be Hausdorff distance between fuzzy numbers, where $[u]_r = [u_1(r),\ u_2(r)],\ [v]_r = [v_1(r),\ v_2(r)].$ The following properties are well known:

$$D(u+w,v+w) = D(u,v),\quad \forall\, u,v,w \in \mathbb{R}_{\mathbb{F}},$$
$$D(k.u,k.v) = |k|\, D(u,v),\quad \forall k \in \mathbb{R},\ u,v \in \mathbb{R}_{\mathbb{F}},$$
$$D(u+v,w+e) = D(u,w) + D(v,e),\quad \forall\, u,v,w,e \in \mathbb{R}_{\mathbb{F}},$$

Where $(\mathbb{R}_{\mathbb{F}}, D)$ is a complete metric space.

Definition 6: (see [8]) A function $f : \mathbb{R} \to \mathbb{R}_{\mathbb{F}}$ is said to be fuzzy continuous function, if f exists for any fixed arbitrary $t_0 \in \mathbb{R}$ and $\varepsilon > 0,\ \delta > 0$ such that $|t - t_0| < \delta \Rightarrow D[f(t),\ f(t_0)] < \varepsilon.$

Definition 7 (see [7]) Let $x, y \in \mathbb{R}_{\mathbb{F}}$, if there exists $z \in \mathbb{R}_{\mathbb{F}}$ such that $x = y + z$, then z is called H-difference of x, y and it is denote by $x \ominus y$. (Note that $x \ominus y \neq x + (-1)y = x - y$).

Definition 8 (see [7]) Let $f : (a,b) \to \mathbb{R}_{\mathbb{F}}$ and $t_0 \in (a,b)$. We say that f is H-differentiable (differentiability in sense of Hukuhara) at t_0, If there exists an element $f'(t_0) \in \mathbb{R}_{\mathbb{F}}$, such that,

1) for all $h > 0$ sufficiently near to zero, $\exists$ $f(t_0 + h) \ominus f(t_0)$, $\exists$ $f(t_0) \ominus f(t_0 - h)$ and the limits,(in the metric D)

$$\lim_{h\to 0^+} \frac{f(t_0+h)\ominus f(t_0)}{h} = \lim_{h\to 0^+} \frac{f(t_0)\ominus f(t_0-h)}{h} = f'(t_0),$$

f is called (1)-differentiable at t_0 or

2) for all $h < 0$ sufficiently near to zero, $\exists$ $f(t_0 + h) \ominus f(t_0)$, $\exists$ $f(t_0) \ominus f(t_0 - h)$ and the limits,

$$\lim_{h\to 0^-} \frac{f(t_0+h)\ominus f(t_0)}{h} = \lim_{h\to 0^-} \frac{f(t_0)\ominus f(t_0-h)}{h} = f'(t_0).$$

f is called (2)-differentiable at t_0.

Theorem (see [7]) Let $f : (a,b) \to \mathbb{R}_{\mathbb{F}}$ be a function denotes by $f(t) = (f_1(t,r), f_2(t,r))$, for each $r \in [0,1]$. Then,

1) if f is (1)-differentiable, then $(f_1(t,r)$ and $f_2(t,r)$ are differentiable functions and

$$f'(t) = (f_1'(t,r), f_2'(t,r)),$$

2) if f is (2)-differentiable, then $f_1(t,r)$ and $f_2(t,r)$ are differentiable functions and $f'(t) = (f_2'(t,r), f_1'(t,r))$.

see [9] .

II. Fuzzy Initial Value Problems

Consider the fuzzy initial value problem

$$y'(x) = f(t, y(t)),\quad y(t_0) = y_0,\quad t \in [t_0\ T].$$

Where f is a fuzzy function with r-level sets of initial value

$[y_0]^r = [y_1(0;r), y_2(0;r)],\ \ r \in [0,\ 1].$

We have $y(t,y) = [y_1(t;r),\ y_2(t;r)]$ and $f(t,y) = [f_1(t,y),\ f_2(t,y)]$ where

$$f_1(t,y) = F[t,\ y_1(t;r),\ y_2(t;r)],$$
$$f_2(t,y) = G[t,\ y_1(t;r),\ y_2(t;r)].$$

By using the extension principle, when $y(t)$ is fuzzy number we have the membership function

$$f(t, y(t))(s) = \sup\{y(t)(\tau) \mid s = f(t,\tau)\},\ s \in \mathbb{R}.$$

It follows that:

$$[f(t,y)]^r = [f_1(t,y;r),\ f_2(t,y;r)],\quad r \in [0\,,1],$$

where

$$f_1(t,y;r) = \min\{f(t,u) \mid u \in [y_1(r),\ y_2(r)]\},$$
$$f_2(t,y;r) = \max\{f(t,u) \mid u \in [y_1(r),\ y_2(r)]\}.$$

Throughout this paper we also consider fuzzy function which is continuous in metric space D. Then the continuity of $f(t, y(t); r)$ guarantees the existence of the definition of $f(t, y(t); r)$ for $t \in [t_0\,, T]$ and $r \in [0\,,1]$. Therefore, the functions F and G are defined.

III. Fuzzy Improved Runge-Kutta Method of Order Four with 3-Stages

Let the exact solution $[Y(t)]^r = [Y_1(t;r),\ Y_2(t;r)]$ which is approximated by.

Based on construction of Improved Runge-Kutta method by Rabiei et al [6], Fuzzy Improved Runge-Kutta method of order four with three stages (FIRK4) is given by:

$$y_1(t_{n+1};r) = y_1(t_n;r) + h(b_1 k_{11}(t_n,\ y(t_n;r)) - b_{-1}k_{-11}(t_{n-1},\ y(t_{n-1};r)) + \sum_{i=2}^{3} b_i\{k_{i1}(t_n,\ y(t_n;r)) - k_{-i1}(t_{n-1},\ y(t_{n-1};r))\}),$$

$$y_2(t_{n+1};r) = y_2(t_n;r) + h(b_1 k_{12}(t_n,\ y(t_n;r)) - b_{-1}k_{-12}(t_{n-1},\ y(t_{n-1};r)) + \sum_{i=2}^{3} b_i\{k_{i2}(t_n,\ y(t_n;r)) - k_{-i2}(t_{n-1},\ y(t_{n-1};r))\}),$$

where

$$k_{11}(t_n,\ y(t_n;r)) = \min\{ f(t_n,u) \,|\, u \in [y_1(t_n;r),\ y_2(t_n;r)]\},$$
$$k_{12}(t_n,\ y(t_n;r)) = \max\{ f(t_n,u) \,|\, u \in [y_1(t_n;r),\ y_2(t_n;r)]\},$$
$$k_{21}(t_n,\ y(t_n;r)) = \min\{ f(t_n + c_2 h,u) \,|\, u \in [z_{11}(t_n,\ y(t_n;r)),\ z_{12}(t_n, y(t_n;r))]\},$$
$$k_{22}(t_n,\ y(t_n;r)) = \max\{ f(t_n + c_2 h,u) \,|\, u \in [z_{11}(t_n,\ y(t_n;r)),\ z_{12}(t_n, y(t_n;r))]\},$$
$$k_{31}(t_n,\ y(t_n;r)) = \min\{ f(t_n + c_3 h,u) \,|\, u \in [z_{21}(t_n,\ y(t_n;r)),\ z_{22}(t_n, y(t_n;r))]\},$$
$$k_{32}(t_n,\ y(t_n;r)) = \max\{ f(t_n + c_3 h,u) \,|\, u \in [z_{21}(t_n,\ y(t_n;r)),\ z_{22}(t_n, y(t_n;r))]\},$$

and

$$z_{11}(t_n,\ y(t_n;r)) = y_1(t_n;r) + ha_{21}k_{11}(t_n,\ y(t_n;r)),\quad z_{12}(t_n,\ y(t_n;r)) = y_2(t_n;r) + ha_{21}k_{12}(t_n,\ y(t_n;r)),$$

$$z_{21}(t_n,\ y(t_n;r)) = y_1(t_n;r) + h\sum_{j=1}^{2} a_{3j}\, k_{j1}(t_n,\ y(t_n;r)),\quad z_{22}(t_n,\ y(t_n;r)) = y_2(t_n;r) + h\sum_{j=1}^{2} a_{3j}\, k_{j2}(t_n,\ y(t_n;r)),$$

we set the coefficients of FIRK4 same as IRK4 which are given as follows (see [6]):

$$c_2 = \frac{31}{60},\ c_3 = \frac{62}{85},\ a_{21} = \frac{31}{60},\ a_{31} = \frac{7502}{24565},\ a_{32} = \frac{10416}{24565},$$
$$b_{-1} = \frac{-157}{23064},\ b_1 = \frac{23221}{23064}\ b_2 = \frac{-1800}{6727},\ b_3 = \frac{122825}{161448}.$$

IV. Convergence Analysis

Here we prove the convergence of FIRK method of order four. By using Taylor series expansion for FIRK method of order $p=4$ with given coefficients, the truncation error T_{n+1}, for FIRK4 up to h^5 is given by

$$\|T_{n+1}\| \le \frac{31}{12240}\sqrt{11924}\, h^5,$$

and $T_{n+1} \approx \dfrac{31}{12240}\sqrt{11924}\, h^5 + O(h^6).$ (1)

Consider the exact solutions $[Y(t)]_r = [Y_1(t;r),\ Y_2(t;r)]$ be approximated by $[y(t)]_r = [y_1(t;r),\ y_2(t;r)]$ and

$$y_1(t_{n+1};r) = y_1(t_n;r) + hF(t_n, y(t_n;r)), \qquad (2)$$

$$y_2(t_{n+1};r) = y_2(t_n;r) + hG(t_n, y(t_n;r)). \qquad (3)$$

Where *F* and *G* are in region *K* define by

$$K = \{(t,u,v) \,|\, 0 \le t \le T,\ -\infty \le u \le +\infty,\ -\infty \le v \le +\infty\}.$$

Based on Fuzzy Improved Runge-Kutta method of order 4 with three stages, we define the *F* and *G* as follows:

$$F(t_n, Y(t_n;r)) = b_1 k_{11}(t_n,\ Y(t_n;r)) - b_{-1}k_{-11}(t_{n-1},\ Y(t_{n-1};r))$$
$$+ \sum_{i=2}^{3} b_i\{k_{i1}(t_n,\ Y(t_n;r)) - k_{-i1}(t_{n-1},\ Y(t_{n-1};r))\},$$

$$G(t_n, Y(t_n;r) = b_1 k_{12}(t_n,\ Y(t_n;r)) - b_{-1}k_{-12}(t_{n-1},\ Y(t_{n-1};r))$$
$$+ \sum_{i=2}^{3} b_i\{k_{i2}(t_n,\ Y(t_n;r)) - k_{-i2}(t_{n-1},\ Y(t_{n-1};r))\}.$$

Following lemmas are used to prove the convergence of FIRK4 method.

Lemma 1 *(see [10]): Let the sequence of numbers* $\{W_n\}_{n=0}^{N}$ *satisfy*

$$|W_{n+1}| \le A|W_n| + B,\quad 0 \le n \le N-1,$$

for some given positive constant A and B, then

$$|W_n| \le A^n |W_0| + B\frac{A^n - 1}{A - 1},\quad 0 \le n \le N.$$

Lemma *(see [10]) Let the sequence of numbers* $\{W_n\}_{n=0}^{N}$ *and* $\{V_n\}_{n=0}^{N}$ *satisfy*

$$|W_{n+1}| \le |W_n| + A\max\{|W_n|, |V_n|\} + B,$$

$$|V_{n+1}| \le |V_n| + A\max\{|W_n|, |V_n|\} + B,$$

for some given positive constants A and B, and denote

$$Un = |W_n| + |V_n|,\quad 0 \le n \le N.$$

then $U_n \le \bar{A}U_0 + \bar{B}\dfrac{\bar{A}^n - 1}{\bar{A} - 1}$, *where* $\bar{A} = 1 + 2A$ *and* $\bar{B} = 2B$.

Theorem 1: *Let F(t,u,v) and G(t,u,v) are in* $C^4(K)$ *and let the partial derivative of F and G be bounded over K, then for arbitrary fixed r ,* $0 \le r \le 1$ *, the approximate solution of (2) and (3) converge to the exact solutions* $Y_1(t;r)$ *and* $Y_2(t;r)$ *uniformly in t.*

Proof: The convergence of method will be proven by

$$\lim_{h\to 0} y_1(t_{n+1};r) = Y_1(t_{n+1};r),$$

$$\lim_{h\to 0} y_2(t_{n+1};r) = Y_2(t_{n+1};r).$$

Fuzzy Improved Runge-Kutta method of order *p=4* is given by

$$Y_1(t_{n+1};r) = Y_1(t_n;r) + hF(t_n, Y(t_n;r)) + T_{n+1}, \quad (4)$$

$$Y_2(t_{n+1};r) = Y_2(t_n;r) + hG(t_n, Y(t_n;r)) + T_{n+1}. \quad (5)$$

where T_{n+1} is given truncation error in equation (1). We define:

$$W_{n+1} = Y_1(t_{n+1};r) - y_1(t_{n+1};r),$$

$$V_{n+1} = Y_2(t_{n+1};r) - y_2(t_{n+1};r).$$

Note for simplification in following equations we set: $t_n = t$, $Y_1(t_n;r) = Y_1$, $Y_2(t_n;r) = Y_2$, $y_1(t_n;r) = y_1$, and $y_2(t_n;r) = y_2$. Hence, by subtracting (2) and (3) from (4) and (5) we have

$$W_{n+1} = W_n + h\{F(t, Y_1, Y_2) - F(t, y_1, y_2)\} + T_{n+1},$$
$$V_{n+1} = V_n + h\{G(t, Y_1, Y_2) - G(t, y_1, y_2)\} + T_{n+1},$$

Therefore

$$|W_{n+1}| \le |W_n| + h|F(t, Y_1, Y_2) - F(t, y_1, y_2)| + \frac{31}{12240}\sqrt{11924}\, h^5$$
$$+O(h^6),$$

$$|V_{n+1}| \le |V_n| + h|G(t, Y_1, Y_2) - G(t, y_1, y_2)| + \frac{31}{12240}\sqrt{11924}\, h^5$$
$$+O(h^6).$$

Four $t \in [t_0, T]$ and $\bar{L} > 0$ is a bound for the partial derivative of *F* and *G* we have

$$|W_{n+1}| \le |W_n| + 2h\bar{L}\max(|W_n|, |V_n|) + \frac{31}{12240}\sqrt{11924}\ h^5$$
$$+O(h^6),$$

$$|V_{n+1}| \le |V_n| + 2h\bar{L}\max(|W_n|, |V_n|) + \frac{31}{12240}\sqrt{11924}\, h^5$$
$$+O(h^6).$$

Using the lemma 1 for $U_n = |W_n| + |V_n|$ with $|U_0| = |W_0| + |V_0|$, we have

$$U_n \le (1+4\bar{L}h)^n U_0 + \frac{(1+4\bar{L}h)^n - 1}{4\bar{L}h}\left(\frac{31}{12240}\sqrt{11924}\, h^5 + O(h^6)\right),$$

where $n = \frac{T}{h}$, therefore we have:

$$U_n \le (1+4\bar{L}h)^n U_0 + \frac{(1+4\bar{L}h)^{\frac{T}{h}} - 1}{4\bar{L}}\left(\frac{31}{12240}\sqrt{11924}\, h^4 + O(h^5)\right).$$

since $W_0 = V_0 = 0$ and using relation $0 \le (1+\omega)^m \le e^{m\omega}$ we have

$$U_n \le \frac{e^{4\bar{L}T} - 1}{4\bar{L}}\left(\frac{31}{12240}\sqrt{11924}\right)h^4 + O(h^5).$$

if $h \to 0$ leads $U_n = (|W_n| + |V_n|) \to 0$ therefore $|W_n| \to 0$ and $|V_n| \to 0$ which complete the proof of Theorem 1.

V. NUMERICAL EXAMPLE

In this section, we solved the fuzzy initial value problems to show the efficiency and accuracy of the proposed methods. Let the exact solution be $[Y(t)]^r = [Y_1(t;r),\ Y_2(t;r)]$ and used to estimate the global error as well as to approximate the starting values of $[y(t_1)]^r = [y_1(t_1;r),\ y_2(t_1;r)]$ at the first step.

We define the

$$error(t_i, y(t_i;r)) = |y(t_i;r) - Y(t_i;r)|$$

We tested the following problems and the numerical results of FIRK4 is given in Tables 1- 2 and Figures 1- 2.

Problem 1: (see [5])

$$y'(t) = y(t)(1-2t), \quad t \ge 0,$$

$$y(0) = [-\frac{\sqrt{1-r}}{2},\ \frac{\sqrt{1-r}}{2}], \quad r \in [0\,1].$$

The exact solution is given by:

$$y(0) = [-\frac{\sqrt{1-r}}{2}e^{t-t^2},\ \frac{\sqrt{1-r}}{2}e^{t-t^2}], \quad r \in [0\,1].$$

Problem 2: (Radioactivity decay model, see [3]).

$$y'(t) = Ay(t) + f,$$

where $y'(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \\ y_3'(t) \\ y_4'(t) \end{bmatrix}, \quad y = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix},$

$$A = \begin{bmatrix} 0 & 0 & -0.4+0.1r & 0 \\ 0.2+0.1r & 0 & 0 & -0.04+0.01r \\ -0.2-0.1r & 0 & 0 & 0 \\ 0 & -0.02-0.01r & 0.4-0.1r & 0 \end{bmatrix},$$

$$f = \begin{bmatrix} 4.9+5r \\ 0 \\ 5.1-0.1r \\ 0 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 995+5r \\ 0 \\ 1005-5r \\ 0 \end{bmatrix}.$$

The exact solutions for $r = 1$ are given by:

$$Y_1(t;r) = Y_3(t;r) = \frac{50}{3} + \frac{2950}{3} e^{\frac{-3}{10}t},$$

$$Y_2(t;r) = Y_4(t;r) = \frac{500}{3} - \frac{29500}{27} e^{\frac{-3}{10}t} + \frac{2500}{27} e^{\frac{-3}{100}t}.$$

TABLE 1: Numerical results at $t_N = 1$, $N = 10$ for problem 1

r	FIRK4		FRK3	
	y_1	y_2	y_1	y_2
0	6.45×10^{-7}	6.45×10^{-7}	2.31×10^{-5}	2.31×10^{-5}
0.1	6.12×10^{-7}	6.12×10^{-7}	2.19×10^{-5}	2.19×10^{-5}
0.2	5.77×10^{-7}	5.77×10^{-7}	2.07×10^{-5}	2.07×10^{-5}
0.3	5.40×10^{-7}	5.40×10^{-7}	1.93×10^{-5}	1.93×10^{-5}
0.4	5.00×10^{-7}	5.00×10^{-7}	1.79×10^{-5}	1.79×10^{-5}
0.5	4.56×10^{-7}	4.56×10^{-7}	1.63×10^{-5}	1.63×10^{-5}
0.6	4.08×10^{-7}	4.08×10^{-7}	1.46×10^{-5}	1.46×10^{-5}
0.7	3.53×10^{-7}	3.53×10^{-7}	1.28×10^{-5}	1.28×10^{-5}
0.8	2.88×10^{-7}	2.88×10^{-7}	1.03×10^{-5}	1.03×10^{-5}
0.9	2.04×10^{-7}	2.04×10^{-7}	7.32×10^{-6}	7.32×10^{-6}
1.0	0	0	0	0

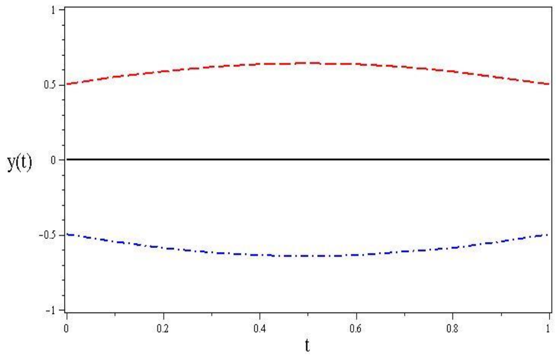

FIGURE 1: The approximated solution of $y_1(t)$ and $y_2(t)$ (solid line) and exact solution (points) with $h = 0.1$, $t \in [0\ 1]$ for problem 1.

TABLE 2: Numerical results of $y_1 = y_3$ and $y_2 = y_4$ with $h = 0.05$, $r = 1$ for Problem 2

t	FIRK4		FRK3	
	y_1	y_2	y_1	y_2
0	0.0	0.0	0.0	0.0
0.5	2.76×10^{-7}	3.06×10^{-7}	1.77×10^{-5}	1.97×10^{-5}
1	4.75×10^{-7}	5.27×10^{-7}	3.06×10^{-5}	3.40×10^{-5}
1.5	6.13×10^{-7}	6.81×10^{-7}	3.95×10^{-5}	4.39×10^{-5}
2	7.03×10^{-7}	7.81×10^{-7}	4.53×10^{-5}	5.04×10^{-5}
2.5	7.57×10^{-7}	7.41×10^{-7}	4.88×10^{-5}	5.42×10^{-5}
3	6.24×10^{-7}	8.68×10^{-7}	5.04×10^{-5}	5.60×10^{-5}

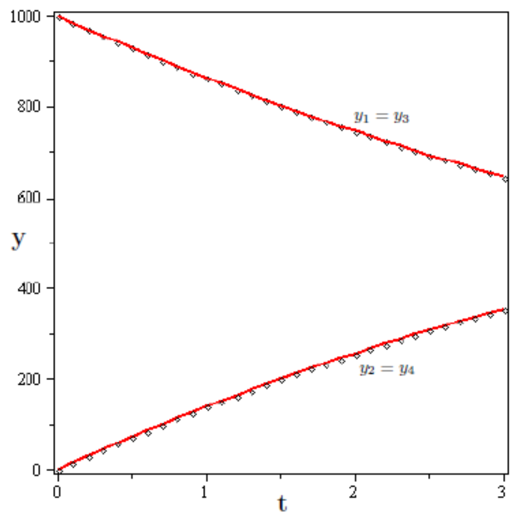

FIGURE 2: The approximated solution of $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$ (solid line) and exact solution (points) with $h = 0.1$, $r = 1$ for problem 2.

VI. CONCLUSION

For tested problems 1 and 2 , the maximum global error of FIRK4 cpmpared with fuzzy Runge-kutta method of order three with 3 stages (FRK3) derived by Dormand [11] are given in Tables 1 and 2. Note that FRK3 is based on original coefficient of classical third order three stages Runge-kutta method which is given in [6]. The numerical results show that FIRK4 with same number of stages gives high error accuracy. Also Figures 1 and 2 show the curve of approximated solution compared with the exact solution and we can see that the approximated solution by FIRK4 almost tends to the exact solution which indicates the accuracy of method .

In this paper we developed Fuzzy Improved Runge-Kutta methods for solving first order fuzzy differential equations. The scheme is two step in nature and is based on the Improved Runge-Kutta method for solving ordinary differential equations. The method of order four with three stages is proposed. Numerical results show that fuzzy Improved Runge-Kutta methods with high error accuracy are efficient for solving first order fuzzy differential equations.

ACKNOWLEDGMENT

This article is financed by University Putra Malaysia using FRGS grant with project number 5524421 at Department of Mathematics, Faculty of Science, Universiti Putra Malaysia.

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