

Probability Problems in Prospect Appraisal

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Abstract: Prospect evaluators are always concerned of the following two problems associated with a prospect: one is the oil discovery probability, the other is the field size distribution for the prospect. These are the two basic exploration parameters which are necessary for formulating different kinds of exploration strategies.

The exploration estimation parameters are the subjective geological judgement of the analysis; however, by the Bayes' rule, the latest new objective information can be incorporated into the original estimation, therefore, the original subjective judgement can be revised systematically and the revised estimation will be closer to the fact in the future.

This paper will discuss the oil discovery probability of a prospect based on the subjective theory of probability. Then it will show how to use the Bayes' rule in revising the field size distribution, estimated by Monte-Carlo simulation, with a speculative prospect.

INTRODUCTION

Prospect evaluators are always concerned with the following two problems associated with a prospect: one is the oil discovery probability, the other is the field size distribution for the prospect. These are the two basic exploration parameters which are necessary for formulating different kinds of exploration strategies.

The exploration estimation parameters are the subjective geological judgement of the analyst; however, by the Bayes' rule, the latest new objective information can be incorporated into the original estimation, therefore, the original subjective judgement can be revised systematically and the revised estimation will be closer to the fact in the future. On the multi-well program, Grayson (1962) used the primary drilling result and Bayes' rule to modify the oil discovery probability that is subjectively estimated before the drilling. In the famous book with the title "Decision Analysis For Petroleum Exploration", Newendrop (1975) wrote a special chapter to illustrate how to use the Bayes' rule in the decision to purchase imperfect information. On the "Acquisition of Information-economic considerations", Ricks (1986) used the Bayes' rule to revise the subjective probabilities assessed for each geological factors of the prospect as new information is obtained for that attribute. Obviously, the functions of Bayes' rule in the petroleum exploration are very powerful and comprehensive.

This paper will discuss the oil discovery probability of a prospect based on the subjective theory of probability. Then it will show how to use the Bayes' rule in revising both the field size distribution, estimated by Monte-Carlo simulation, and the oil discovery probability for a speculative prospect.

SUBJECTIVE THEORY OF PROBABILITY AND BAYES' RULE

According to the subjective theory of probability, the probability of an event is the degree of confidence which the people believe that the event will occur, expressed by a relative number. For instance, the raining probability tomorrow is the degree of confidence which the people believe that it will rain tomorrow. Obviously, the subjective judgement on the possibility of an event to occur is defined as the probability the event will occur. From definition, if an event will occur certainly, then the degree of confidence of its occurrence is one ; it means that the event's probability is 1; on the other hand, if an event will certainly not occur, then the degree of confidence of its occurrence is zero; it means that the event's probability is 0 ; therefore, the range of probability is [0,1].

In a sample space, using A criteria to classify the space into A and Non-A and using B criteria to classify the same space into B₁, B₂, ..., B_s. If P(A), the probability of A, do not equal to zero, then

$$P(B_j|A) = \frac{P(B_j) \times P(A|B_j)}{\sum_{j=1}^s P(B_j) \times P(A|B_j)} \quad j = 1, 2, \dots, s$$

this is the Bayes' rule

P(B_j) is known as a priori probability, it is the original subjective probability assessed for the B_j event. P(B_j|A) is called a posteriori probability, it is a conditional probability of event B_j after the event A has occurred and is being calculated to replace the P(B_j). P(A|B_j) is the conditional probability of event A after the event B_j has occurred and can be calculated by available information. The process in which using P(A|B_j) to revise P(B_j) is known as Bayesian Analysis.

OIL DISCOVERY PROBABILITY

Rose(1987) analysed the exploration activities in the United States from 1977 to 1978. He found that the success rate for all United States new-field-wild-cats during the period was 16%, however, only about 2% of the total new-field wild-cats made the discovery in which the recoverable reserve exceeding 1 million BOE. Therefore, it is believed that the real success rate for commercial new-field wild-cats probably is not much higher than about 5%. Nevertheless, most new-field wildcat prospects are assigned discovery probability of 10 to 30%.

The discovery probability for wildcat well in the range of 0.1 to 0.3 reflects the fact that oil discovery has a rather low probability. Bourdair, etc (1985) pointed out that it is hard to calibrate those corresponding probabilities on psychological grounds. This explains why appraiser seldom assess the probability directly. Instead, they evaluate the probabilities of the geological factors which affect the oil accumulation, in the prospect. Such as

- ...presence of source rock
- ...migration and timing
- ...reservoir, porosity, recovery
- ...trapping and sealing

According to the geological data of the prospect, the degree of confidence which each of these factors will occur in the prospect is assessed and expressed by a relative number, then the subjective oil discovery probability is obtained from multiplying these numbers by each other, however, the geological data quality influences the assessment, therefore, the degree of confidence on the data quality must account for the probability estimation. For example, suppose the subjective probability for each of these factors is 0.8,0.7,0.6 and 0.5 and the degree of confidence on the data quality is 0.75, then the subjective oil discovery probability for the prospect is

$$0.8 \times 0.7 \times 0.6 \times 0.5 \times 0.75 \\ = 0.126 = 12.6\%$$

FIELD SIZE DISTRIBUTION

Confronting the imperfect geological data and for appraising the oil accumulation ability of a prospect, the assessor always sets up different geological hypotheses to simulate all possible field size distributions and the final desired distribution is obtained by averaging these simulated distributions. For instance, in order to include all possible geological situations the evaluator may set up three triangular distribution data for the area of accumulation. If the distribution of all other parameters, such as net pay, are kept unchanged, then there are three simulated field size distributions. The average distribution of these three distributions is the expected distribution.

So far, only part of the job of the field size estimation in the prospect analysis has been discussed. Any latest objective new information about the field size must be incorporated into the estimation, to make the estimation more meaningful. For example, what kind of modification on the estimated distribution should be done if the first well drilled on the prospect confirms that the recoverable oil in the prospect probably is 10 million barrels? It means that with the average field size distribution, the evaluator must analyse WHAT IF problems of all the possible outcomes of the expected distribution. The Bayes' rule has been used to deal with this problem very successfully.

Suppose a prospect has only two mutually exclusive geological interpretations: case A and case B. The former interpretation is more optimistic in the area of closure than the latter and the interpreter believes that both have the same probability of occurring. Table 1 is the list of the relative frequency of recoverable reserve estimated by Monteimulation for the two cases.

TABLE 1
RECOVERABLE RESERVE DISTRIBUTIONS

LEVEL OF THE RESERVE	Unit: million barrels							
	1-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
CASE A	6	7	10	15	12	6	3	1
CASE B	12	18	10	8	5	4	2	1

Since the two cases are equally likely, the final expected field size distribution is obtained by summing the two relative frequencies together first and then divide the sum by 2 for each reserve level. The result is shown on the table 2 in terms of relative percentage.

TABLE 2
THE DESIRED FIELD SIZE DISTRIBUTION

	Unit: million barrels							
LEVEL OF THE RESERVE	1-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
RELATIVE %	15	21	17	19	14	8	4	2

(mean value of this distribution is 14.8)

If the first well drilled on the prospect probably hits 23 million barrels of recoverable oil, then what are the new probabilities for the two geological interpretations and what is the new expected field size distribution for the prospect?

In the example, the total number of simulations is 60 and the relative frequencies of the reserve in the range of 20 to 25 are 12 and 5 for case A and case B respectively, therefore, the conditional probabilities of 23 million barrels in the two cases are $P(23|A)=12/60=0.20$ and $P(23|B)=5/60=0.08$ respectively. These are the conditional probabilities to be used in the Bayesian analysis.

Substitute them into the Bayes' rule and use $P(A|23)$ and $P(B|23)$ to denote the new probabilities for the two cases ; then

$$\text{By} \quad P(E_j|F) = \frac{P(E_j) \times P(F|E_j)}{\sum_{j=1}^s P(E_j) \times P(F|E_j)} \quad j = 1, 2, \dots, s$$

$$\begin{aligned} \text{So} \quad P(A|23) &= \frac{P(A) \times P(23|A)}{P(A) \times P(23|A) + P(B) \times P(23|B)} \\ &= \frac{0.5 \times 0.20}{0.5 \times 0.20 + 0.5 \times 0.08} \\ &= 0.714 \end{aligned}$$

$$\begin{aligned} P(B|23) &= \frac{P(B) \times P(23|B)}{P(A) \times P(23|A) + P(B) \times P(23|B)} \\ &= \frac{0.5 \times 0.08}{0.5 \times 0.20 + 0.5 \times 0.08} \\ &= 0.286 \end{aligned}$$

Having gotten these revised probabilities which are the weighting factor for calculating expected field size distribution, then the new expected field size distribution can be obtained

from the original estimations directly. For instance the new relative frequency of the reserve ranging from 20 to 25 million barrels is $(12 \times 0.714 + 5 \times 0.286)/60 = 0.17$. The revised expected field size distribution is shown on table 3.

The above analysis shows that the new reserve information obtained from the first well has been used to modify the geological interpretations:

1. the probability of case A increases from 0.5 to 0.714, meanwhile, the probability of case B decreases from 0.5 to 0.286 and the possibility of case A is 2.5 times over case B.
2. the most reserve level of the distribution shifted from the range 5-10 million barrels

TABLE 3
THE REVISED FIELD SIZE DISTRIBUTION

	Unit: million barrels							
LEVEL OF THE RESERVE	1-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
RELATIVE %	13	16	16	22	17	9	5	2

(mean value of this distribution is 16.1)

to 15-20 million barrels and the mean values of the distributions increased from 14.8 million barrels to 16.1 million barrels.

FINDING A GIVEN LEVEL OF RESERVE

The actual probability of finding a given level of reserve in the prospect can be obtained from the oil discovery probability and the expected field size distribution for the prospect. In the previous example, the actual probabilities for all given level of reserves are shown on table 4 .

TABLE 4
THE PROBABILITIES FOR THE GIVEN LEVEL OF RESERVES

OIL DISCOVERY PROBABILITY	LEVEL OF RESERVES	RELATIVE FREQUENCY		ACTUAL FREQUENCY	
		PRIORI	POSTERIORI	PRIORI	POSTERIORI
0.126	1-5	0.15	0.13	0.019	0.016
	5-10	0.21	0.16	0.026	0.020
	10-15	0.17	0.16	0.021	0.020
	15-20	0.19	0.22	0.024	0.028
	20-25	0.14	0.17	0.018	0.022
	25-30	0.08	0.09	0.010	0.011
	30-35	0.04	0.05	0.005	0.006
	35-40	0.02	0.02	0.003	0.003
		1.00	1.00	0.126	0.126

The data on the table shows that the Bayesian analysis has revised the actual probabilities for each reserve level in the optimistic direction and successfully reflected the optimistic viewpoint resulting from the outcome of the first wildcat well .

REVISING OIL DISCOVERY PROBABILITY

Based on the data at hand, the subjective probability in the previous example for trapping and sealing factors is 0.6. Now suppose the new information about the factors is provided and the reliability of that information is perfect. It means that when the trapping & sealing factors do exist in the prospect, 100% of the time the information will identify them as trapping and sealing factors. Therefore, the new subjective probability for trapping and sealing factors is 1.0 and the oil discovery probability for the prospect will increase to 21%. However, most of the time, the information about the subsurface is imperfect. Suppose the reliability of that information is only 85%, what is the new subjective probability for trapping and sealing factors?

This is a typical Bayesian analysis problem and the following table method for solving the problem is suggested by Newendrop (1976).

POSSIBLE STATES OF NATURE	ORIGINAL RISK ESTIMATES	RELIABILITY OF INFORMATION	JOINT PROBABILITIES	REVISED RISK ESTIMATES
TRAPPING & SEALING	0.6	0.85	0.51	$\frac{0.51}{0.57} = 0.89$
NO TRAPPING & SEALING	0.4	0.15	0.06	$\frac{0.06}{0.57} = 0.11$
	1.0	1.0	0.57	1.0

It shows that if the reliability of the new information about trapping & sealing factors is 85%, then the subjective probability for the factors will increase from 60% to 89%. Therefore, the oil discovery probability for the prospect will increase from 12.6% to 18.69%.

The above procedure should be used in the same manner to revise the subjective probabilities assessed for the other four factors, in order to obtain new oil discovery probability for the prospect when any new information is obtained for them.

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APPENDIX

Stochastic Decision Tree

The discussed method is used to solve the stochastic decision tree which is associated with purchase imperfect information. In the case of figure

1. the imperfect information is the reliability of seismic interpretation. If the reliability of seismic interpretation is R and the result of the seismic program shows that the oil is there, then the oil discovery probability increases to

$$\frac{P \times R}{P \times R + (1-P) \times (1-R)}$$

otherwise, the oil discovery probability falls to

$$\frac{P \times (1-R)}{P \times (1-R) + (1-P) \times R}$$

For instance, in the case of figure 1, if P=0.15 and R=0.8, then P'=0.414 (the result of

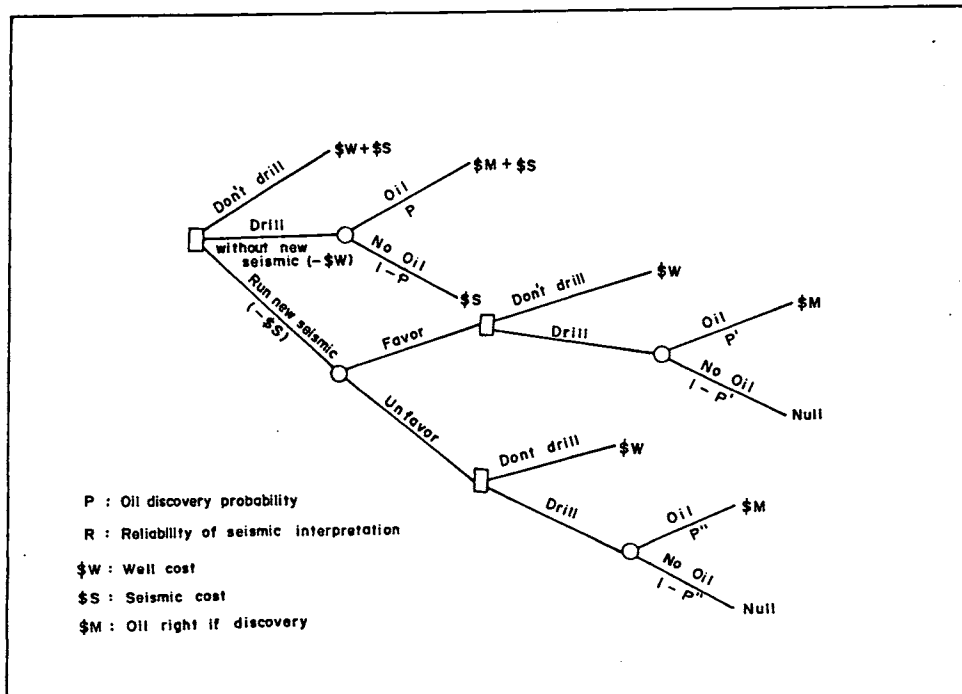


Figure 1: The stochastic decision tree

seismic program shows that oil is there, then the oil discovery probability increases to 0.414) and $P''=0.04$ (the result of seismic program shows that oil is not there, then the oil discovery probability falls to 0.04).

Let maximization of one's expected asset position be the decision rule and if the other decision parameters such as \$W, \$S and \$M have been assigned then the stochastic decision tree can be solved. The following BASIC program is very helpful to solve the tree. X, Y and Z are the expected asset value for the three cases; Don't drill. Drill without new seismic and run new seismic before drilling.

```

10  CLEAR
20  INPUT "PROBABILITY =";P
30  INPUT "RELIABILITY =";R
40  INPUT "WELL COST $=";W
50  INPUT "SEISMIC COST $=";S
60  INPUT "OIL RIGHT $=";M
70  P1=(P*R) / ((P*R)+(1-P)*(1-R))
80  P2=(P*(1-R) / (P*(1-R)+(1-P)*R))
90  Y1=P1*M
100 IF W>=Y1 THEN Y1=W
110 Y2=Y1*((P*R)+(1-P)*(1-R))
120 Z1=P2*M
130 IF W>=Z1 THEN Z1=W
140 Z2=Z1*(P*(1-R)+(1-P)*R)
160 Z=Z2+Y2-S
170 Y=P*(M+S)+(1-P)*S-W
180 X=W+S
190 C=X
200 IF Y>=C THEN C=Y
210 IF Z>=C THEN C=Z
220 PRINT X;Y;Z;C

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