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Application of finite difference eikonal solver for traveltime computation in forward modeling and migration

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Abstract: Traveltime is one of the propagating wave's components. As the wave propagates further, the traveltime increases. It can be computed by solving wave equation of the ray path or the eikonal wave equation. Accurate method of computing traveltimes will give a significant impact on enhancing the output of seismic forward modeling and migration. In seismic forward modeling, computation of the wave's traveltime locally by ray tracing method leads to low resolution of the resulting seismic image, especially when the subsurface is having a complex geology. However, computing the wave's traveltime with a gridding scheme by finite difference methods able to overcomes the problem. This paper aims to discuss the ability of ray tracing and fast marching method of finite difference in obtaining a seismic image that have more similarity with its subsurface model. We illustrated the results of the traveltime computation by both methods in form of ray path projection and wavefront. We employed these methods in forward modeling and compared both resulting seismic images. Seismic migration is executed as a part of quality control (QC). We used a synthetic velocity model which based on a part of Malay Basin geology structure. Our findings shows that the seismic images produced by the application of fast marching finite difference method has better resolution than ray tracing method especially on deeper part of subsurface model.

Keywords: Reflection, traveltime, forward modeling, ray tracing, eikonal equation, gridding scheme, fast marching, finite difference method, migration

INTRODUCTION

In general, seismic reflection data in the seismogram are acquired from the responses of the seismic wave. The study of seismic wave propagation through the subsurface body can be done either in forward modeling or in real seismic acquisition. Nanda (2016) states that reflection event is the most major phenomenon in imaging the subsurface. In oil and gas, the application of seismic reflection method is generally for exploring subsurface layers for 100 meters up to 5000 meters of depth (Kruk, 2001). In terms of basic seismic processing and imaging, the seismic image that we observed is associated with some projecting-ray line components such as traveltime, amplitude and frequency. According to the discussions by Bording *et al.* (1987) and Lines & Newrick (2004), these components can be derived into some physical properties such as density and velocity using specific methods. In addition, the characteristics of these components may vary depending on the rock properties themselves. Sowers & Boyd (2019) explained that density or mineral alignments able to result in variation of the arrival time of the reflected seismic waves. Schmitt (2015) mentioned that porosity and fluid saturations also give impact on the speed of the reflected seismic waves.

Computation of traveltimes is needed in seismic modeling, processing, and imaging methods. In seismology field for example, an accurate seismic traveltimes prediction method is necessary for their reflection processing, pinpointing earthquake source location and seismic tomography (Rawlinson & Sambridge, 2005) including the processing of seismic reflection profiles, earthquake location, and seismic tomography at a variety of scales. In this paper, we present two seismic applications of a recently developed grid-based numerical scheme for tracking the evolution of monotonically advancing interfaces, via finite-difference solution of the eikonal equation, known as the fast marching method (FMM). Also, traveltime field is required as the input data for Kirchhoff migration and velocity analysis process (Alkhalifah & Fomel, 2010; Zhang & Bording, 2011). It shows that, the application of a reliable traveltime computation algorithm is very crucial in these seismic areas to produce a seismic image with better resolution. Two main approaches that can be used to calculate the seismic wave traveltimes from source to the receiver, which are the traditional ray tracing method and finite difference approximation to the eikonal equation (Perez & Bancroft, 2001; Alkhalifah & Fomel, 2010; Alashloo & Ghosh, 2017). Further discussion of these approaches is done on the next section.

Seismic forward modeling can be described as the use of geological model of earth (e.g., in form of velocity or

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density properties) for practicing mathematical algorithmic simulation. This technique allows geoscientist to study and approximate how seismic waves propagate through the subsurface model from the recorded seismogram. Moreover, the algorithmic simulation also might be useful for geoscientist in designing acquisition survey and interpretation (Chapman, 2004). For example, based on the area or depth of targeted subsurface, suitable seismic acquisition parameters such as number of shots, number of receivers, distance between shots or receivers can be set to obtain seismic image with good signal to noise ratio. Consequently, based on the good responses upon testing on the synthetic data, same parameters can be applied towards real acquisition. Besides designing acquisition parameters, we might want to test other mathematical forward modeling algorithm such as traveltime calculation algorithm of propagating seismic waves towards specific geological condition. If the resulting outcomes match to a degree of satisfaction or accuracy, the applied algorithm is reasonable towards that synthetic model and can be applied on the real subsurface too. For instance, if we want to study seismic responses of a salt body or gas body area, the synthetic model itself must include these geological features too. However, if the outcomes obtained is bad, the algorithm is said to be not suitable and need to be modified or replace with more reliable one.

As mentioned previously, traveltimes field is needed for the seismic migration process. Seismic migration is an algorithm which is very synonym in seismic imaging to reconstruct subsurface image (Jones, 2018). It is an inversion operation that focusing on rearrangement of seismic information elements such as traveltimes so that the reflections are returned to their actual position and collapse the refraction hyperbola effects. The migration process can be done for pre-stack or post-stack, 2D or 3D data in time or depth domain. A velocity field is needed as the input for the migration to correct mispositioned reflectors. This data can be obtained by executing velocity analysis. Through the velocity analysis process, geophysicists derive velocity model from the recorded traveltimes. Results from normal moveout (NMO) velocity analysis can be useful when we are dealing with simple subsurface condition. However, subsurface body is not usually simple. Complex geological features will make the seismic data processing and imaging itself to be more challenging and more accurate computing algorithm is needed. Finally, a reliable and accurate seismic image is generated to enable geological interpretation take part and estimation of material properties distribution such as oil and gas is obtained through the inversion process. In this paper, we discussed on the reliability of ray tracing method and fast marching method of finite difference eikonal solver applications towards calculating traveltimes of propagating seismic waves in the forward modeling. We also discussed on the effect of both ray tracing and the finite difference methods when being utilized in forward modeling and seismic migration.

VELOCITY AND TRAVELTIMES

Basic mathematical expression of the traveltime is shown by the simple velocity, $\boldsymbol{v}$ distance, $\boldsymbol{d}$ and time, $\boldsymbol{t}$ relationship below:

$$v = d/t \qquad (1)$$

Based on equation (1) above, velocity of a moving object is the product of total distances it travelled over the time taken. This concept also is applicable for seismic waves. Seismic waves that propagate at a distance through a homogeneously isotropic medium will move at a constant speed or velocity with the travelling times varies. In seismic reflection imaging, the traveltime-depth is described as two-way vertical traveltime, $\boldsymbol{t_{twt}}$ since the waves propagate down first from source and propagate upward to the receiver later (Claerbout, 2010). The different arrival time of the waves propagating from the source back to the receiver is used to figure out the subsurface properties. The equation can be expressed as below:

$$t_{twt} = 2z/v \qquad (2)$$

where $\boldsymbol{z}$ is the depth or the vertical distance from surface. As mentioned earlier, traveltime is one of the key components in seismic data analysis for imaging structure and velocity. It also being utilized to deduce information of the rocks, especially about the physical characteristics of the beds. For example, waves travel slower in a less density medium and faster in a compressed medium. Based on equation (2), traveltime is inversely proportional to velocity. As the velocity increase, shorter time taken is recorded. In addition, traveltime is directly proportional to the depth. The deeper subsurface layer causes the wave to take a longer time to reach the receivers on the surface.

Eikonal equation

In isotropic medium, the traveltimes from a fixed source is governed by the eikonal equation which has been derived from Pythagoras theorem 2500 years ago (Robinson & Clark, 2017). Eikonal equation, which has been developed by Sir William Rowan Hamilton about 15 decades ago is a theoretical ray approximation towards the scalar wave equation (Wilkins, 2020). This equation is obtained by finding plane harmonic solutions and employ the high frequency approximation of ray theory (Lecomte *et al.*, 2000). In other words, the eikonal equation controlled the traveltimes field for a fixed source in a heterogeneous (Alkhalifah & Fomel, 2010). In the 3D case, the eikonal equation can be written as:

$$(\partial t/\partial x)^2 + (\partial t/\partial y)^2 + (\partial t/\partial z)^2 = (1/v)^2 = s^2 \qquad (3)$$

The eikonal equation above is referred to Rickett & Fomel (1999), Rawlinson *et al*. (2008) and Alkhalifah &

Fomel (2010) which is derived from the basic equation (1). Furthermore, this eikonal equations classified as a non-linear partial differential equation for traveltime. ***(x, y, z)*** are Cartesian coordinate axes, ***t*** is the traveltime and ***s*** is the slowness which is reciprocal to velocity. Ray and finite difference among known methods used to solve the eikonal equation (Kraaijpoel, 2003).

Ray tracing method

Ray tracing is the procedure of tracing the ray path by solving the Hamilton's ray equation. The Hamilton's ray equation describes how a motion system evolves with time, which in our case, the propagation of ray. According to Robinson & Douze (1985), two Hamilton's ray equation is the product of using the eikonal equation in ray tracing. This condition treats rays as equivalent to the characteristic curves of the Hamiltonian. In simpler words, if we manage to solve Hamilton's equation, the ray path can be traced, hence the procedure is called as ray tracing. Ray tracing uses the assumption of the wave traveling like a ray through the shortest path (Fermat's principle) in the model and change direction when encountering velocity and density difference. Although the result is accurate and regularly used, Vidale (1990) and Perez & Bancroft (2001) have mentioned that this method is computationally intensive, unable to solve shadow zones and sometimes overlooks the shortest ray path. Moreover, rays tend to cross each other when propagating in a complex velocity model (Zhang & Bording, 2011). Besides accuracy diminish upon dealing with greater complexity of the subsurface geology, the presence of large sources and receivers number also leads to a large amount of time consumption in the computing process of ray tracing method (Rawlinson & Sambridge, 2005).

Fast marching method of finite difference eikonal solver

Finite difference eikonal solver was introduced by Vidale (1988) for calculating traveltimes on a 2D or 3D velocity model (Zhang et *al*., 2005). Recently, it becomes more popular especially in predicting traveltimes for complex subsurface. Instead of calculating wave propagation locally as in ray tracing, finite difference offers to solve the eikonal equation over the whole earth model by dividing the velocity field into gridding scheme. Although finite difference eikonal solver is limited to locate first arrival traveltimes, the computation is exceptionally fast (Rawlinson & Sambridge, 2005; Alashloo & Ghosh, 2017). Hence, make the extraction of traveltimes, ray paths and wavefront geometry for huge number of sources and receivers possible to be done. Fast marching method which has been introduced by Sethian, (1996) is an example of the modern traveltimes computation methods that have been extensively applied in seismology (Rawlinson & Sambridge, 2005; Alkhalifah & Fomel, 2010; Zhang & Bording, 2011; Alashloo & Ghosh, 2017). This method is classified as an efficient and unconditionally stable grid-based eikonal solver (Alkhalifah & Fomel, 2001), and has been adopted by Popovici & Sethian (2002) in their migration of reflection profiles. Moreover, Alashloo & Ghosh (2017) employed fast marching method for the Vertical Transverse Isotropic (VTI) concept and use the results in the Kirchhoff depth migration algorithm. Kirchhoff migration is known as an algorithm that sum up the diffraction hyperbola to its apex and repositioned the apparent location of the reflector to the true location by using the integral form of wave equation (Smitha *et al.*, 2016).

Fast marching method retrieves the traveltimes of the grid points by solving the eikonal equation (Hui *et al.*, 2017). The calculation of the traveltimes to every grid point can be expressed in terms of how we track the evolution of propagating seismic wavefront from source through all over the medium (Rawlinson & Sambridge, 2005) by directly imitating the propagating wavefront. During solving the eikonal equation for the first arrival traveltimes field, it was discussed by Rawlinson & Sambridge (2005) that the difficulty of finite-difference methods faces is the wavefront gradient discontinuity which exist when arriving information is discarded after the wavefront self-intersects (multi-pathing). However, fast marching method handle this difficulty by implementing an entropy condition. Sethian & Popovici (1999) explained that information can only be lost (with multi-pathing) or saved (absence of multi-pathing) when wavefront expanded since it can cross a point once only. Strict implementation of this condition yields to the stability of fast marching method. The entropy which satisfies upwind scheme stated is written as:

$$\begin{bmatrix} \max\left(D_a^{-x}T, -D_b^{+x}T, 0\right)^2 \\ +\max\left(D_c^{-y}T, -D_d^{+y}T, 0\right)^2 \\ +\max\left(D_e^{-z}T, -D_f^{+z}T, 0\right)^2 \end{bmatrix}_{i,j,k}^{\frac{1}{2}} = s_{i,j,k} \qquad (4)$$

where ***(i, j, k)*** are Cartesian grid increment variables in ***(x, y, z)***. The order of accuracy of the upwind finite-difference operator used in every six cases is defined by the integer variables ***a, b, c, d, e*** and ***f***. Let us take the first two upwind operators for $D^{-x}T_i$:

$$D_1^{-x}T_i = \frac{T_i - T_{i-1}}{\delta x}$$
$$D_2^{-x}T_i = \frac{3T_i - 4T_{i-1} + T_{i-2}}{2\partial x} \qquad (5)$$

where ∂**x** is the spacing of the grid in **x.** Availability of upwind traveltimes and maximum order allowed will determine which operator is used in equation (4). D_1 operator only used by first-order schemes. Meanwhile, D_2 operator use a second-order scheme. However, in case the absent of T_{i-2} (e.g., near a source point), D_2 will revert to D_1. In this paper, we used second-order scheme as it able to reduce errors as well as maintaining stability, efficiency and simplicity (Rickett & Fomel, 1999). To sum up, the updating process of traveltimes upon grid point not only is done by solving the eikonal equation (3), but also required implementation of entropy equation (4).

In fast marching method, narrow band technique is introduced for constructing the expansion of wavefront from the source to the medium's body. The narrow band is built around the traveltime wavefront as illustrated in Figure 1. To improve computing efficiency, heapsort is used to select the lowest traveltime grid value within the narrow band and stored it at the top of the heap (Alashloo & Ghosh, 2017; Hui *et al*., 2017). The wavefront always propagates using the minimum of known traveltime value within the heap to calculate the next unknown traveltimes. The use of the heapsort algorithm indicates that fast marching method has a heap operation count, which is O(NlogN) where N is the overall number grid points (Rickett & Fomel, 1999; Rawlinson & Sambridge, 2005; Alashloo & Ghosh, 2017). In this case, the overall performance of fast marching method highly affected by the scales of computational cost and grid size.

The algorithm divides the grid points into three group as shown in Figure 1. They are labelled as *Alive* points, *NarrowBand* points and *FarAway* points. First, at source location where the wave begins to propagate, the grid points are put as *Alive* and time is set to 0, which corresponding to distance = 0. Second, the algorithm will find the point having minimum traveltime value within the *NarrowBand* points nearby and calculate it. Then, the point is set as *Alive*. The *Alive* points stay behind the wavefront where traveltime is already calculated, accepted and will not undergo further change. *FarAway* points are the points ahead of the wavefront are labelled as *FarAway*. For this group, the traveltime values are remained untouched, not calculated, and unfixed. On third step, the neighboring points of *FarAway* side are updated as the next *NarrowBand*. Second and third steps continue to loop until all points are covered by the wavefront or in other words, changed to *Alive*.

NUMERICAL TEST ON SYNTHETIC MODEL

Synthetic model

The 2D synthetic velocity as in Figure 2 above is designed using Tesseral Pro software, developed by Tesseral Technologies ("Tesseral Pro," 2021). The model has a total distance of 4.6 km with 1.5 km depth. Each layer is set to have difference velocity value and representing one of subsurface conditions of Malay Basin. The first layer having 1.5 km/s P-wave velocity indicates the water layer while the rest are sedimentary rock layers having velocity increase with depth. On left and right side, we have some channels. The basement is set to have 4 km/s P-wave velocity. Our purpose of setting this kind of complex structures is to observe the ability of both ray tracing and fast marching method to calculate traveltimes of the wavefront propagating throughout the whole model. In general, wave will travel at low speed through a less dense medium and travel faster

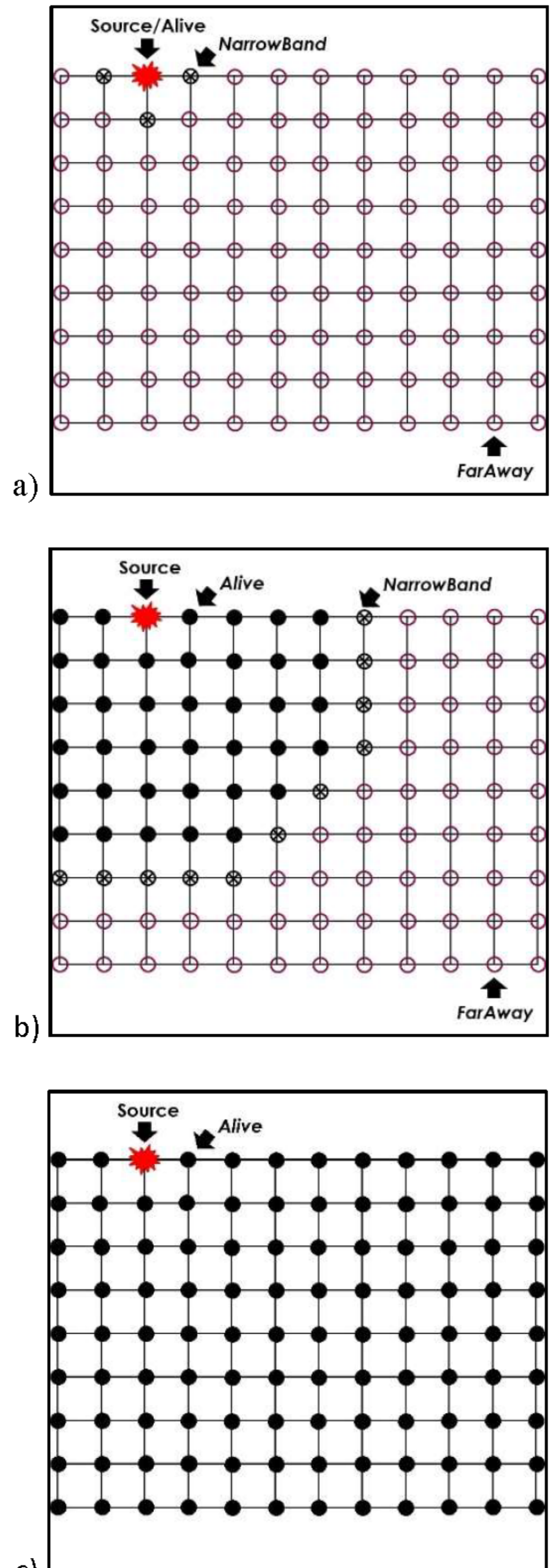

Figure 1: Illustration of complete process of updating traveltime values of every grid point of fast marching method.

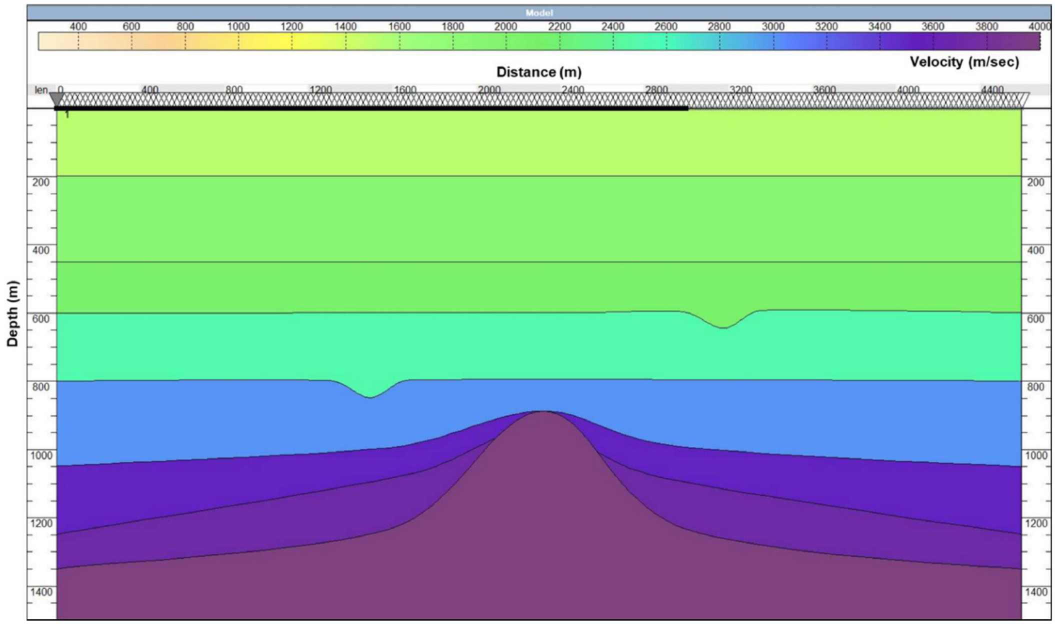

Figure 2: Synthetic model, viewed in Tesseral Pro software.

through a denser medium. Wave also will be reflected as they travel from a less dense to a denser medium. Furthermore, having higher velocity indicates the rock also having higher density measurements. Due to the limitation of computing power and time, the grids are set to have total of 4 m x 4 m in size for the forward modeling codes to be practically executable upon it.

Traveltime calculation using ray tracing and fast marching method

The first arrival traveltimes is calculated using the velocity model for sources located at x = 0.5 km, x = 1.5 km, x = 2.5 km, and x = 4.0 km. We executed the traveltime calculations and show the results in Madagascar seismic package software ("Madagascar," 2018). It is an open-source software, written in python codes. Traveltimes field produced by ray tracing is compared with the output of fast marching method of finite difference as shown in Figure 3 and Figure 4. Results produced by ray tracing is illustrated by the ray path projections meanwhile the result from fast marching finite difference method is demonstrated by the first arrival traveltimes contour (wavefront). As mentioned before, ray tracing was unable to calculate traveltime on shadow zone and it can be seen clearly in Figure 3. Shadow zone is the area where rays did not pass through due to angle of incidence and its reflectivity to the surface. Furthermore, we noticed that shadow zones appear when the rays' path are blocked by the high velocity – high density anticlinal structure in the middle of the model. This condition shows that at certain source location, some data might not be recorded at the shadow zones area. In other words, we can say that some important structures will not being recorded correctly by the seismic wave and may leads to inaccurate seismic image later. Meanwhile, at different source locations as in Figure 4, fast marching finite difference algorithm able to calculate the traveltimes over the whole area in the model even in the high complexity geological condition. As the layers change in velocity and density values, traveltime contours expand by following the boundaries throughout the medium. This demonstrates the advantage of calculation of traveltime with a gridding scheme. This shows that the method is more reliable in obtaining traveltimes data throughout model per source location. For example, traveltime data covered by ray tracing method is ~ 40% less than fast marching method at source location = 0.5 km, 1.5 km, and 4.0 km. Meanwhile, at source location = 2.5 km, ray tracing covered ~ 10% less than fast marching. In summary, fast marching finite difference method provides better coverage compared to ray tracing.

Forward modeling

In seismic forward modeling, the data acquisition survey is simulated for the marine synthetic model. Table 1 shows all parameters applied to the acquisition simulation in the Tesseral Pro software.

The moving receivers with source imitate marine seismic acquisition like a ship towing a source and receiver cable moving through a total of 4.6 kms area. The long receiver cable spread of 3.0 km is set to ensure better coverage up to the deeper part of the model subsurface. Forward modeling is done to obtain seismic image from different modeling procedure which is using the finite difference and ray tracing.

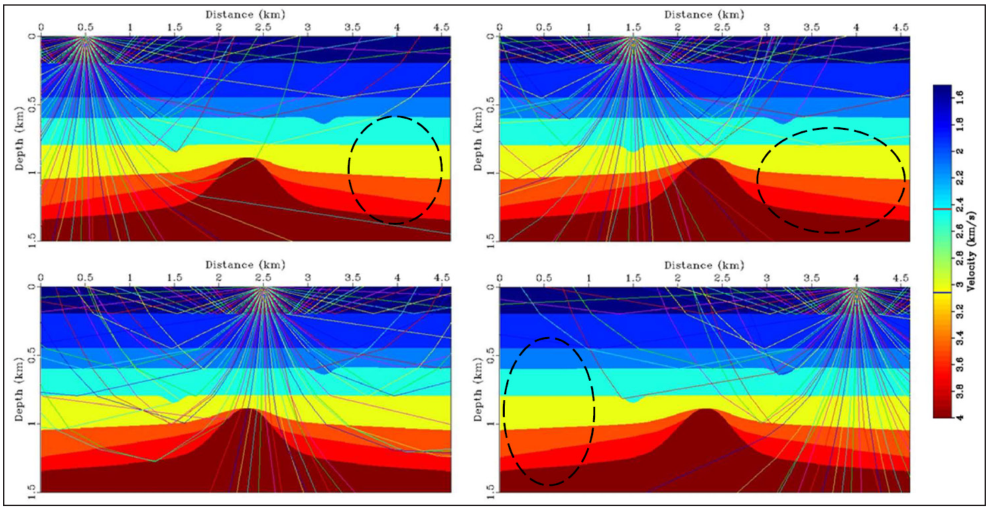

Figure 3: Traveltime calculation using ray tracing. The black dotted circles are the shadow zones (area uncovered by the ray path).

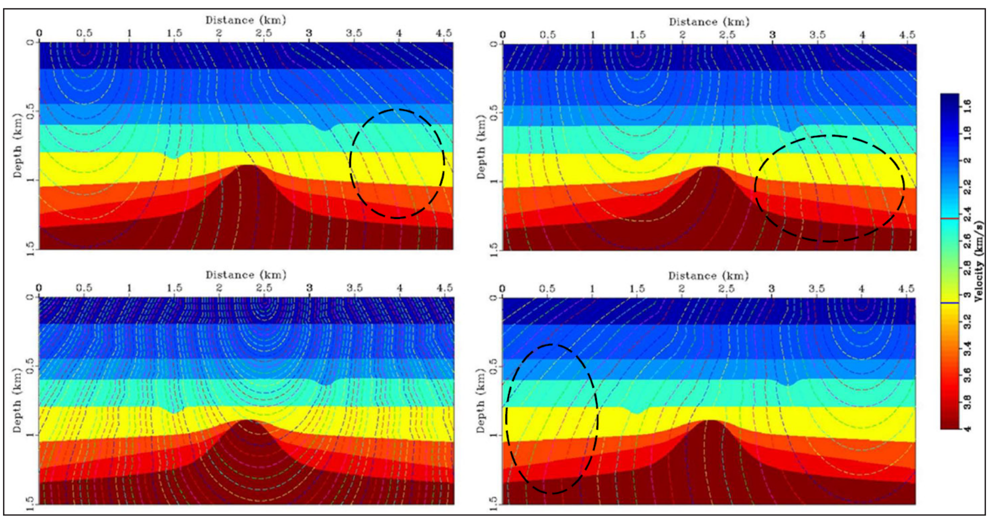

Figure 4: Traveltime calculation using fast marching method. The shadow zones have been covered by the wavefronts.

Table 1: Parameters applied on the seismic acquisition simulation in Tesseral Pro software.

Acquisition scheme	Move receivers with source
Source frequency	15 Hz
Wavelet	Ricker
Shot number	185
Shot interval	25 meters
Receiver number	240
Receiver interval	12.5 meter
Number of samples	1500
Sampling rate	2 milliseconds
Recoding time	3 seconds

As discussed before, finite difference method able to calculate traveltime better than ray tracing and provided better coverage of the subsurface image. We have tested the reliability of finite difference to calculate traveltime using fast marching method and compared it with ray tracing. We continued the application of finite difference and ray tracing method in forward modeling process to observe the resolution of resulting seismic images. Figure 5 and Figure 6 show stacked seismic image of ray tracing and finite difference procedure respectively after NMO correction. It is clearly can be observed that forward modeling using finite difference procedure produced a better resolution of seismic image than ray tracing. This happens because, on single source location, gridding scheme of finite difference algorithm manage to calculate traveltimes for whole medium. In other words, it gives better recorded data coverage than ray tracing algorithm. If we combined or stacked the traveltime field from all different source locations, the image from the output of finite difference method is having 95% resemblance towards the original subsurface model (Figure 2). However, ray tracing only shows 60% resemblance. For example, flat subsurface layers (0 km to 1.0 km) were clearly seen for both methods.

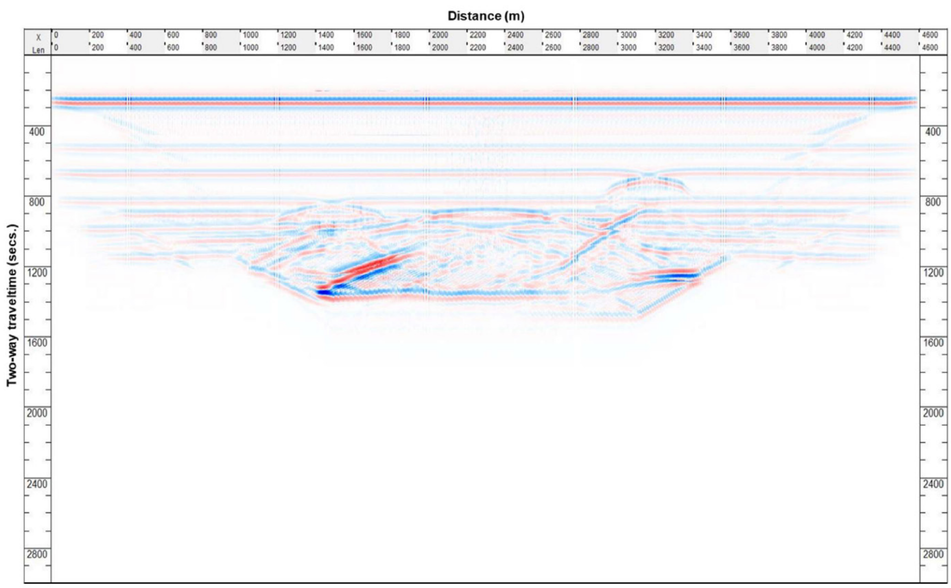

Figure 5: Stacked seismic image from ray tracing procedure after NMO correction.

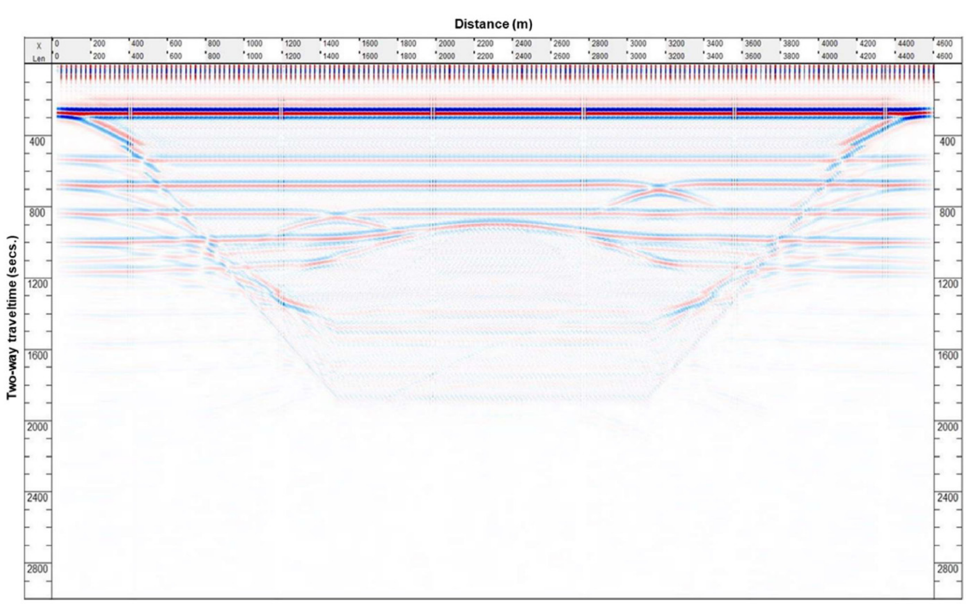

Figure 6: Stacked seismic image from finite different procedure after NMO correction.

But the image quality diminished for ray tracing method when the subsurface changes to more dipping conditions. The channels structure and boundary of anticlinal bedrock can be clearly seen with excellent continuity by the finite difference method. It is proven that ray tracing unable to handle such complex geology condition. Furthermore, the same happened for seismic image after applying Kirchhoff migration. After removing the effect of diffraction and repositioning the depth of reflectors, seismic image of finite difference method (Figure 8) has better reflectors continuity and more obvious boundary than ray tracing (Figure 7). The application of migration upon both seismic images is done as a part of quality control (QC).

CONCLUSION

In conclusion, correct traveltime calculation method shows a vital part in improving the output of seismic forward modeling and imaging. Based on the experiment towards our data, calculation of seismic traveltimes with a gridding scheme of finite difference algorithm is 90% more accurate in term of its coverage upon the whole subsurface model. This method manages to outcome the disadvantages of ray tracing in solving the shadow zone area. Its application in forward modeling also shows huge differences in term of resolution and quality of the output image. The boundary of complex structure especially on deeper part is imaged

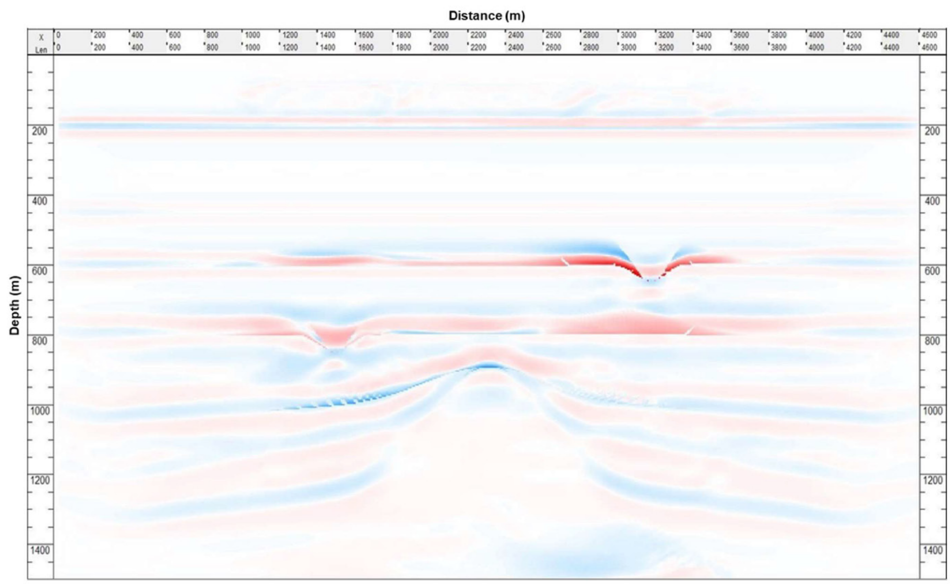

Figure 7: Pre-stack Kirchhoff migrated seismic image in depth domain of ray tracing procedure.

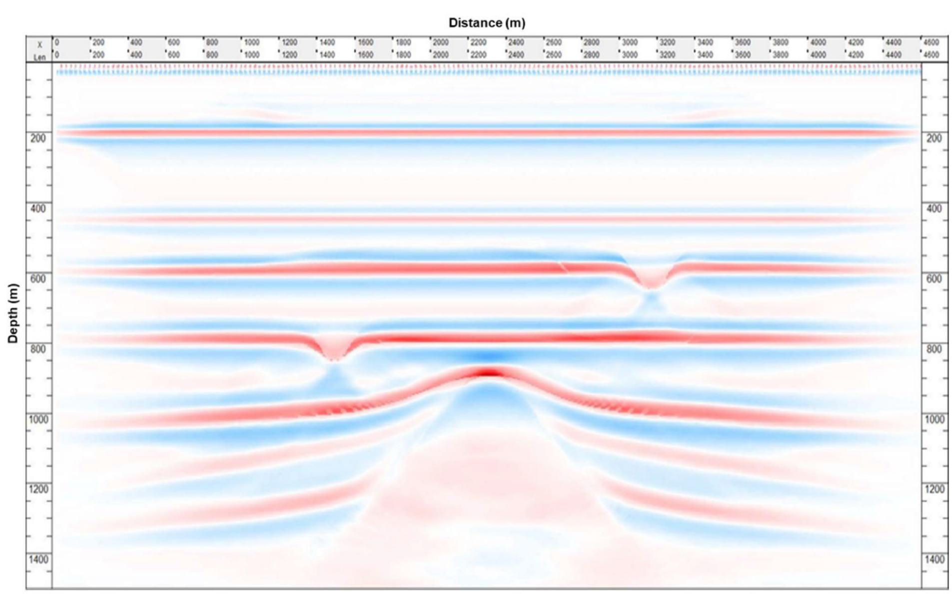

Figure 8: Pre-stack Kirchhoff migrated seismic image in depth domain of finite different procedure.

properly by finite difference algorithm. Seismic migration is one example of QC which is good to justify the reliability of the application of different traveltimes calculation techniques. By having more accurate traveltimes, better seismic imaging can be achieved.

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AUTHOR CONTRIBUTIONS

AMM designed and performed the experiment. SYMA was involved in computation framework of travel time values using the grids scheme. NNAA designed the synthetic model by setting up the suitable velocity values. SYMA, ARMA and AHAL improved the results analysis and interpretation by giving critical feedback. AMM wrote the manuscript, assisted by NNAA.

CONFLICT OF INTEREST

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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